Sensitivity of Thiele's Vented Loudspeaker Enclosure Alignments to Parameter Variations*†

D. B. KEELE, JR.

Electro-Voice, Inc., Buchanan, Mich. 49107

Additional information on the use and application of Thiele's alignments for the vented loudspeaker cabinet is presented. A rewritten alignment table which has all the frequency terms normalized to the speaker resonance frequency is included. Computer-run frequency responses for all the alignments are displayed along with a new fourth-order Chebyshev alignment beyond no. 9. Variations and sensitivity functions for the vented cabinet output with respect to various system parameters (both Thiele system parameters and driver physical parameters) are derived and plotted.

GLOSSARY OF SYMBOLS

B	magnetic flux density in driver air gap
C	electrical capacitance
C_{AB}	acoustic compliance of air in enclosure
C_{AS}	acoustic compliance of driver suspension
C_{MS}	mechanical compliance of driver suspension ($=C_{AS}/S_D^2$)
e_{in}	electrical voltage at input of driver
$E(s)$	response transfer function relating driver input voltage to system acoustic output sound-pressure level in far field
f	frequency, in Hz
f_3	alignment (-3dB) cutoff frequency
f_{aux}	corner frequency of auxiliary filter
f_B	Helmholtz resonance frequency of vented box
f_H, f_L	frequencies of higher and lower peaks of driving point impedance of driver mounted in vented enclosure
f_{pk}	frequency at peak boost for Thiele auxiliary second-order high-pass filter
f_S	resonance frequency of driver in free air
g	normalized corner frequency of auxiliary filter ($=f_{aux}/f_S$)
h	vented enclosure tuning ratio ($=f_B/f_S$)
$H(s)$	voltage transfer function of Thiele auxiliary filter
K	a constant
l	length of voice-coil conductor in magnetic field
L	electrical inductance
M_{MS}	mechanical mass of driver diaphragm assembly including air load
$M(w)$	system function used in explanation of sensitivity function
p_0	acoustic sound pressure in far field of system
Q	ratio of reactance to resistance (series circuit) or resistance to reactance (parallel circuit)
Q_{aux}	Q of Thiele auxiliary filter
Q_B	total enclosure Q at f_B due to all enclosure losses
Q_E	Q of driver at f_S considering system electrical resistance (R_g+R_E) only
Q_{ES}	Q of driver at f_S considering electrical resistance R_E only

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† The major part of this research was done while the author was with the Electronic Media Department, Brigham Young University, Provo, Utah 84601.

Q_M — Q of driver at f_S considering system nonelectrical resistances only

Q_T — total Q of driver at f_S including all system resistances

Q_{TS} — total Q of driver at f_S due to all driver resistances ($=Q_T$ if $R_g=0$)

R — electrical resistance

R_E — dc resistance of driver voice coil

R_g — output resistance of source or amplifier

s — complex frequency variable ($=\sigma+j\omega$)

s_N — complex frequency variable normalized to ω_S ($=s/\omega_S=j\omega/\omega_S$)

S_D — effective projected surface area of driver diaphragm

$S_x^{M}(w)$ — sensitivity function of dependent system function $M(w)$ with respect to independent variable $x\left(=\dfrac{x}{M(w)}\dfrac{\partial M(w)}{\partial x}\right)$

V_{AS} — volume of air having same acoustic compliance as driver suspension ($=\rho_0 c^2 C_{AS}$)

V_B — net internal volume of enclosure

x — independent variable used in explanation of sensitivity function

X_{aux} — reciprocal of Thiele auxiliary filter Q_{aux} ($=1/Q_{\text{aux}}$)

Y — auxiliary-filter constant used and defined by Thiele [1, p. 388]

α — ratio of acoustic compliance of driver suspension to acoustic compliance of air in enclosure ($=C_{AS}/C_{AB}=V_{AS}/V_B$)

ρ_0 — density of air ($=1.18$ kg/m^3)

ω — radian frequency variable, in radians/second ($=2\pi f$)

ω_{aux} — radian corner frequency of auxiliary frequency

ω_B — radian resonance frequency of box ($=2\pi f_B$)

ω_N — normalized radian frequency ($=\omega/\omega_S$)

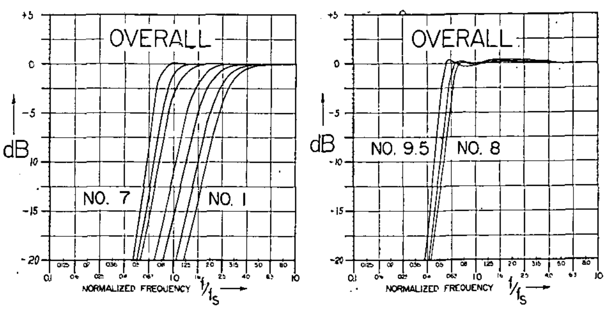
ω_S — radian resonance frequency of driver ($=2\pi f_S$)

ω_0 — corner frequency of passive auxiliary filter

INTRODUCTION: Ever since the republication of Thiele's discourse on vented-box design and analysis [1], this author has been intrigued by Thiele's application of transfer-function filter synthesis techniques to the design of direct-radiator loudspeaker systems. The author being primarily an electrical engineer, recognized that once a system's input–output behavior has been properly characterized by the use of a transfer function, the system can be analyzed (frequency, phase, and impulse response, etc.), synthesized, and manipulated on paper or by computer to make the job of the designer much easier.

Thiele categorized the vented-box loudspeaker system with a fourth-order high-pass transfer function (Eqs. (1), (2)) whose coefficients are written in terms of easily measured system and driver parameters (V_B, f_B, f_S, Q_T, V_{AS}, called Thiele system parameters).[1] He displayed an alignment table (a rewritten version of which appears in this paper as Table I) in which he tabulated his system parameters to synthesize several different types of fourth-, fifth-, and sixth-order high-pass electroacoustic frequency response functions (the fifth- and sixth-order responses require the use of an auxiliary high-pass filter). The author was moved by a desire to see the details of these alignment responses; hence the computer derived response graphs shown in this paper.

[1] J. E. Benson [6, 1968], in a much more detailed analysis of a generalized direct-radiator loudspeaker system, has written a complex fifth-order transfer function which in reduced form describes the operation of several different types of direct-radiator systems (closed-box, lossy vented-box, lossless vented-box (Thiele's model), passive-radiator, infinite baffle etc.).



a. b.

Fig. 1. Magnitude frequency response of Thiele's fourth-order alignments (reading from right to left). **a.** Nos. 1 to 7. **b.** Nos. 8 to 9, including author's C4 alignment, called no. 9.5.

The author was also interested in knowing what effect changes in system parameters would have on the frequency response of the vented-box loudspeaker system. Questions such as "What happens to the response when the box is tuned to a higher or lower frequency than optimum?" "How sensitive is the cabinet output to shifts in driver suspension compliance?" or "The box volume is smaller than designed, how will the frequency response be affected?" need to be answered. The author applied the powerful filter-analysis technique known as parameter-sensitivity analysis to attempt to answer these questions about the vented-box system.

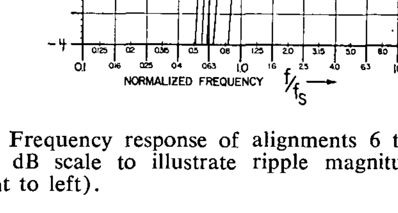


Fig. 2. Frequency response of alignments 6 to 9.5 on an expanded dB scale to illustrate ripple magnitude (reading from right to left).

REWRITTEN ALIGNMENT TABLE

To facilitate identification of this paper's computer-run alignment frequency responses and to simplify design procedures, Thiele's alignment table [1, p. 388] is reproduced here in modified form (Table I). All the frequency terms have been normalized to the driver free-air resonance frequency instead of the alignment (-3 dB) cutoff frequency. Also, the Thiele column giving the ratio C_{AS}/C_{AB} has been changed to the reciprocal value $C_{AB}/C_{AS} = V_B/V_{AS}$ to make the value proportional to the box volume. These changes orient the alignment table toward the designer who starts with a specific driver and then determines the box parameters for a particular alignment. Thiele's original table favored a person who designs drivers, given a specific cabinet size and low-frequency response specification.

The constant listed for the auxiliary filter, X_{aux}, is the reciprocal of the required filter Q ($X_{\text{aux}} = 1/Q = \sqrt{2 + Y}$), where Y is the parameter used by Thiele and defined in [1, Eq. (53), p. 388]. Also included in the new table is a listing for the frequencies f_L and f_H, the impedance peak frequencies for the driving point impedance of the speaker mounted in the vented cabinet. The tabulated values of f_L and f_H help the designer to check the tuning of the completed system.

ALIGNMENT RESPONSES

To ease the computer programming of the Thiele vented-cabinet responses, Thiele's Eq. (19) [1, p. 386] (the operational form of the transfer relationship between the speaker input voltage and the sound-pressure output of the speaker mounted in its cabinet) was rewritten to conform to the following canonical form for transfer functions. After appropriate substitutions and manipulations, this equation becomes

$$E(s) = \frac{p_0(s)}{e_{\text{in}}(s)} \cdot \frac{e_{\text{in}}(\infty)}{p_0(\infty)} = \frac{s^4}{s^4 + (\omega_S/Q_T)s^3 + [\omega_B^2 + \omega_S^2(1+\alpha)]s^2 + (\omega_B^2\omega_S/Q_T)s + \omega_S^2\omega_B^2} \quad (1)$$

where

$s = \sigma + j\omega$ complex variable

$\omega_S = 2\pi f_S$ fundamental resonance frequency of loudspeaker, in rad/s

$\omega_B = 2\pi f_B$ cabinet resonance frequency, in rad/s

$\alpha = \dfrac{C_{AS}}{C_{AB}} = \dfrac{V_{AS}}{V_B}$ ratio of loudspeaker suspension compliance to box compliance (or alternately, ratio of loudspeaker compliance equivalent volume to box volume)

Q_T total Q of driver at f_S including all system resistances.

This transfer function has been normalized so that the response for $\omega >> \omega_3$ is unity. Hence it does not show the dependence of the pass-band level on the Thiele parameters according to Thiele's efficiency equation [1, Eq. (77), p. 472]. The Q_T used in Eq. (1) includes all the losses attributable to the driver (suspension losses, voice coil I^2R losses, etc.) plus amplifier losses. In the derivation of $E(s)$, Thiele assumes a high Q for the cabinet mesh ($Q_B > 30$).

To normalize the response function to the speaker resonance frequency, a substitution of $s_N = s/\omega_S = j(\omega/\omega_S) = j\omega_N$ and $h = \omega_B/\omega_S$ is made in Eq. (1), yielding

$$E(s_N) = \frac{s_N^4}{s_N^4 + (1/Q_T)s_N^3 + (1 + h^2 + \alpha)s_N^2 + (h^2/Q_T)s_N + h^2} \quad (2)$$

Table I. Rewritten alignment data

	Alignment Details				Box Design				Auxiliary Circuits				Impedance Peak Frequencies		
	No.	Type	K	Ripple (dB)	f_3/f_S	f_B/f_S	V_B/V_{AS}	Q_T	f_{aux}/f_S	X_{aux}	Peak Lift (dB)	f_{pk}/f_S	f_L/f_S	f_H/f_S	f_H/f_L
Quasi Third Order	1	QB_3	—		2.68	2.000	0.0954	0.180	—	—	—	—	0.5127	3.901	7.61
	2	QB_3	—		2.28	1.730	0.1337	0.209	—	—	—	—	0.5161	3.346	6.48
	3	QB_3	—		1.77	1.420	0.2242	0.259	—	—	—	—	0.5282	2.681	5.075
	4	QB_3	—		1.45	1.230	0.3390	0.303	—	—	—	—	0.5406	2.273	4.205
Fourth Order	5	B_4	1.0	—	1.000	1.000	0.7072	0.383	—	—	—	—	0.5688	1.758	3.09
	6	C_4	0.8	—	0.867	0.927	0.9479	0.415	—	—	—	—	0.5771	1.607	2.78
	7	C_4	0.6	0.13	0.729	0.829	1.372	0.466	—	—	—	—	0.5741	1.445	2.52
	8	C_4	—	0.25	0.641	0.757	1.790	0.518	—	—	—	—	0.5615	1.348	2.40
	9	C_4	—	0.55	0.600	0.716	2.062	0.557	—	—	—	—	0.5499	1.302	2.37
	9.5	C_4	—	1.52	0.520	0.638	2.60	0.625	—	—	—	—	0.5166	1.235	2.39
Fifth Order	10	B_5	1.0	—	1.000	1.000	1.000	0.447	1.000	—	—	—	0.6180	1.618	2.62
	11	C_5	0.7	—	0.852	0.912	1.715	0.545	1.218	—	—	—	0.6451	1.414	2.19
	12	C_5	0.4	0.25	0.724	0.814	3.663	0.810	1.810	—	—	—	0.6666	1.221	1.83
	13	C_5	0.355	0.5	0.704	0.798	4.405	0.924	2.06	—	—	—	0.6713	1.189	1.77
	14	C_5	0.278	1.0	0.685	0.781	5.236	1.102	2.47	—	—	—	0.6725	1.161	1.73
Sixth Order Class I	15	B_6	1.0	—	1.000	1.000	0.366	0.299	1.000	0.518	+ 6.0	1.070	0.4710	2.123	4.51
	16	C_6	0.8	—	0.850	0.979	0.429	0.317	0.858	0.420	+ 7.7	0.901	0.4864	2.013	4.14
	17	C_6	0.6	—	0.698	0.931	0.552	0.348	0.712	0.318	+10.1	0.733	0.5032	1.850	3.68
	18	C_6	0.5	—	0.620	0.888	0.662	0.371	0.639	0.265	+11.6	0.651	0.5094	1.743	3.42
	19	C_6	0.414	0.1	0.554	0.841	0.800	0.399	0.576	0.2215	+13.2	0.576	0.5123	1.642	3.20
Sixth Order Class II	20	B_6	1.0	—	1.000	1.000	1.000	0.408	1.000	1.414	—	—	0.6180	1.618	2.62
	21	C_6	0.8	—	0.844	0.885	1.385	0.431	0.928	1.250	+ 0.2	1.992	0.6051	1.463	2.42
	22	C_6	0.6	—	0.677	0.738	2.000	0.461	0.819	1.029	+ 1.1	1.181	0.5611	1.315	2.34
	23	C_6	0.5	—	0.592	0.656	2.415	0.484	0.752	0.895	+ 1.9	0.965	0.5235	1.253	2.39
	24	C_6	0.414	0.1	0.520	0.584	2.832	0.513	0.681	0.766	+ 3.0	0.806	0.4832	1.208	2.50
	25	C_6	0.268	0.6	0.404	0.461	3.623	0.616	0.553	0.518	+ 6.0	0.594	0.4000	1.153	2.88
Sixth Order Class III	26	B_6	1.0	—	1.000	1.000	1.366	0.518	1.000	1.931	—	—	0.6599	1.515	2.30
	27	C_6	0.268	0.6	0.778	0.854	9.091	1.503	2.12	1.414	—	—	0.7605	1.123	1.48
	28	QB_3	—	—	0.952	0.971	0.529	0.328	1.028	—	+ 6.0	0	0.5140	1.889	3.68

where

$h = \omega_B/\omega_S = f_B/f_S$ normalized frequency variable which is the tuning ratio between box and speaker resonance frequencies.

In like manner, the normalized transfer functions for the auxiliary filters are derived.

First Order (Alignments 10–14)

$$H_1(s_N) = \frac{s_N}{s_N + g} \tag{3}$$

where

$g = \frac{f_{\text{aux}}}{f_S} = \frac{f_3}{f_S} \cdot \frac{f_{\text{aux}}}{f_3} = \frac{\omega_{\text{aux}}}{\omega_S}$ tuning ratio between auxiliary-filter corner frequency and speaker resonance frequency

$\omega_{\text{aux}} = 2\pi f_{\text{aux}}$ corner frequency of filter, in rad/s

$\omega_3 = 2\pi f_3$ 3-dB down frequency of overall response (speaker plus filter), in rad/s.

Second Order (Alignments 15–27)

$$H_2(s_N) = \frac{s_N^2}{s_N^2 + X_{\text{aux}}\, g\, s_N + g^2} \tag{4}$$

where

$g = \frac{f_{\text{aux}}}{f_S}$

$X_{\text{aux}} = 1/Q_{\text{aux}} = \sqrt{Y+2}$ a constant which is the reciprocal of the filter required Q

$Y =$ auxiliary-filter constant defined by Thiele and appearing in his alignment table.

First Order (Alignment 28)

$$H_3(s_N) = \frac{s + 2g}{s + g}. \tag{5}$$

Computations show that this filter is 3 dB down at $\omega = \sqrt{2}\,g$.

The overall responses of the fifth- and sixth-order loudspeaker systems are just the appropriate products of Eqs. (2), (3), (4), and (5), as shown below.

Fifth Order (Alignments 10–14, 28)

$$E_{5\text{th}}(s) = E(s)H_1(s) \text{ or } E(s)H_3(s). \tag{6}$$

Sixth Order (Alignments 15–27)

$$E_{6\text{th}}(s) = E(s)H_2(s). \tag{7}$$

The complete set of computer-run frequency responses are shown in Figs. 1–5. The alignments which require the use of an auxiliary filter have been shown with separate responses for the loudspeaker driver and cabinet, the filter, and the speaker–filter combination. Examination of the speaker-only responses for alignments 12, 13, 14, and 27 (Fig. 4) clearly show why these responses were considered suspect by Thiele [1, p. 389]. Benson [6, pp. 412–460], in a very thorough analysis of different vented-enclosure synthesis procedures, displays a large number of frequency response graphs which show further effects of different parameter combinations.

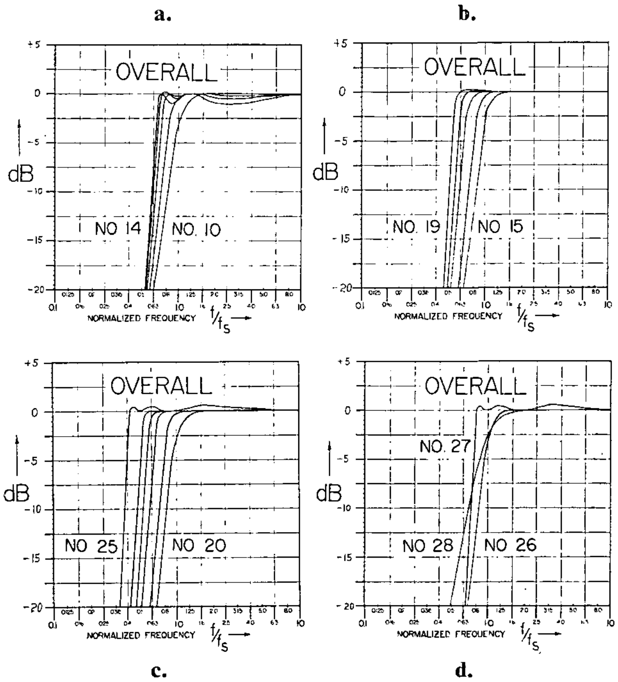


Fig. 3. Overall frequency response (including effects of auxiliary filter) of alignments (reading from right to left). **a.** Nos. 10 to 14 (fifth order). **b.** Nos. 15 to 19 (sixth order, Class I). **c.** Nos. 20 to 25 (sixth order, Class II). **d.** Nos. 26 and 27 (sixth order, Class III) and No. 28 (fifth order which uses a first-order auxiliary lift filter).

NEW RIPPLE VALUES AND C4 ALIGNMENT

As a result of the computer runs, it was noted that the ripple magnitude quoted by Thiele in his alignment table (for the Chebyshev alignments 7, 8, 9) was in excess of the ripple values as determined from the computer responses. For example, Thiele indicates a ripple value of 1.8 dB for alignment 9, but the computer-run response (see Fig. 2) shows a ripple value of about 0.55 dB.[2] For this paper, the author is defining the ripple as the difference between the maxima and minima in the passband in dB [2, pp. 374–375]. The rewritten alignment Table I reflects the new ripple values.

Because of the comparatively small value of ripple for alignment 9, the author was moved to investigate fourth-order alignments with a higher value of ripple (and hence a lower cutoff frequency). Alignment 9.5 is a result of this study (see Figs. 1 and 2).[3] This align-

[2] In private correspondence with Richard Small of the University of Sydney, Australia, Dr. Small indicates that both he and J. E. Benson of Amalgamated Wireless (Australasia) Limited discovered this ripple error also, and that A. Thiele to this day does not know how he managed to go wrong on the calculations for his 1961 paper [1].

[3] At the time this paper was written the author was unaware of the excellent work R. H. Small [5] and J. E. Benson [6] were doing on synthesis of the Chebyshev alignments. Both Small and Benson show results of alignments with up to 3 dB of ripple and Benson indicates how to calculate the parameters for any arbitrary value of ripple.

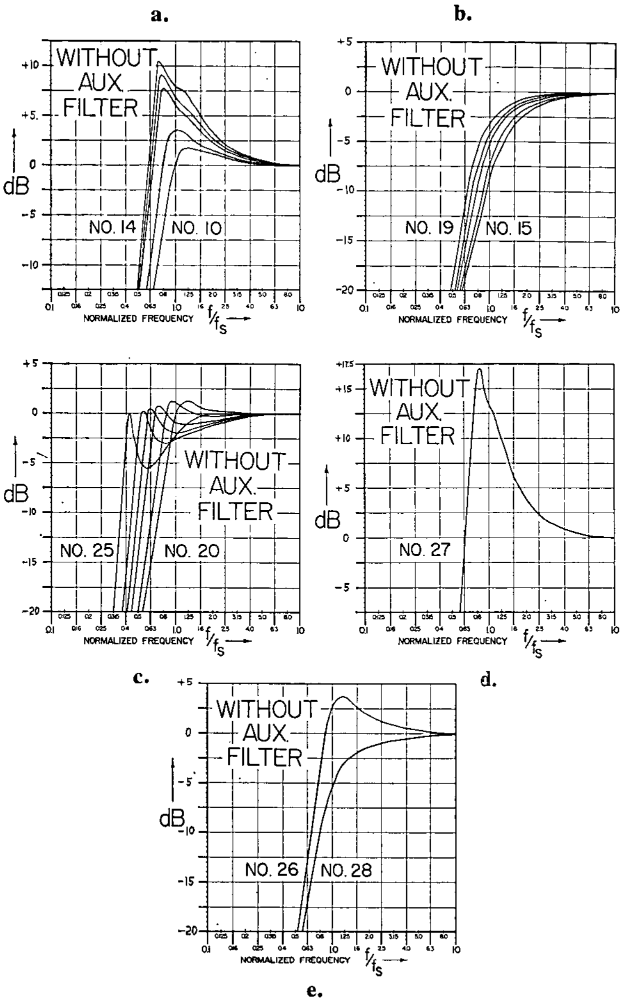


Fig. 4. Frequency response of loudspeaker system without auxiliary filter for alignments (reading from right to left. **a.** Nos. 10 to 14. **b.** Nos. 15 to 19. **c.** Nos. 20 to 25. **d.** No. 27. **e.** Nos. 26 and 28.

ment has a low-frequency cutoff nearly a full octave below the speaker resonance frequency (0.52 f_S), ripple of about 1.5 dB, a required Q_T of 0.625, and requires a volume of 2.6 times speaker compliance equivalent volume.

PERTURBATION OF SYSTEM PARAMETERS

To illustrate the quantitative effects of variations of the system constants on the frequency response, several responses were run with nonoptimum values for the system parameters. The fourth-order alignments 1, 5, and 9 were chosen for this perturbation study.

These variations were performed on two different sets of parameters, 1) the Thiele system parameters (ω_B, ω_S, Q_T, and α), and 2) the fundamental driver physical parameters, (Bl, M_{MS}, and C_{MS}). These two sets of parameters are related through the following equations:

$$\omega_S = \sqrt{\frac{1}{M_{MS}\,C_{MS}}} \tag{8}$$

$$Q_T = \frac{R_E}{(Bl)^2}\sqrt{\frac{M_{MS}}{C_{MS}}} \tag{9}$$

$$\alpha = \frac{C_{AS}}{C_{AB}} = \frac{S_D^2\,C_{MS}}{C_{AB}}. \tag{10}$$

Eq. (9) is an approximation which is derived by assuming no driver mechanical losses and an amplifier source impedance of zero. Variations of both sets of parameters were included in this study to fully describe parameter shifts which would most likely occur in practice.

For a particular variation, all the parameters in the specific parameter set being investigated (either Thiele system parameters or driver physical parameters) were held constant at their correct alignment values except one, which was varied in one-sixth octave steps (ratio of 1.121 to 1 or about 12%) above and below the correct value. The variational responses are shown in Figs. 6–12. It again must be pointed out that all the variational responses have been normalized so that the pass-band level for $\omega >> \omega_3$ is unity (0 dB). Changes in the parameters ω_S, Q_T, V_{AS}, Bl, and M_{MS} all cause changes in the pass-band level according to Thiele's efficiency equations [1, Eqs. (66), (67), p. 471].

The perturbation responses for variations of f_B and Q_T are very similar to the variational responses illustrated in Novak [3, pp. 9–10]. The writer's observations concerning the parameter perturbations will be withheld until after the next section which displays the alignment parameter sensitivity functions.

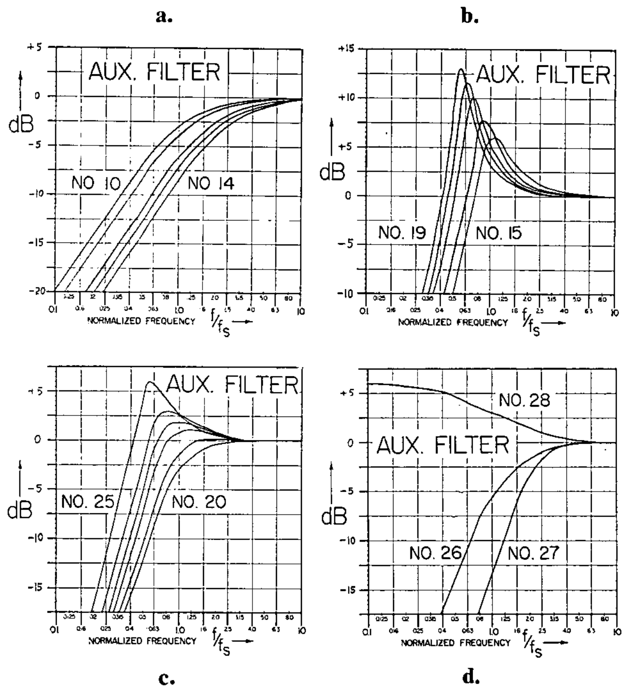


Fig. 5. Frequency response of auxiliary filter for alignments. **a.** Nos. 10 to 14 (left to right). **b.** Nos. 15 to 19 (right to left). **c.** Nos. 20 to 25 (right to left). **d.** Nos. 26 to 28.

SENSITIVITY FUNCTIONS

To show the quantitative effects of system parameter changes on the frequency response, the sensitivity functions for the magnitude of Eqs. (1) and (2) were derived for both sets of parameters described in the previous section.

Sensitivity is a measure of how some characteristic of a system changes when certain system parameters are perturbed [4, p. 461]. The sensitivity of a system function $M(w)$ with respect to a parameter x is defined in [4, p. 462]:

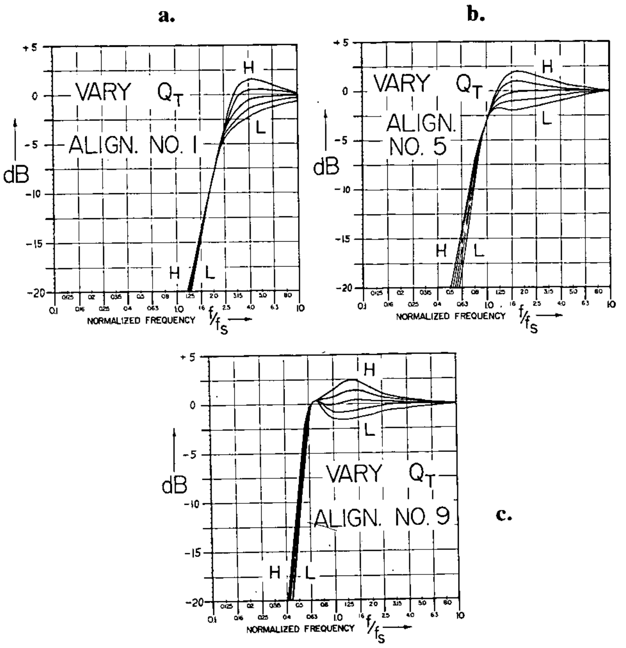


Fig. 6. Effect of variation of system Q_T on response of alignments. **a.** No. 1. **b.** No. 5. **c.** No. 9. Step factors of 0.794, 0.891, 1.000, 1.122, and 1.259 above and below the correct Q_T are illustrated. Thiele system parameters f_B, f_S, V_B, and V_{AS} are held constant for this variation.

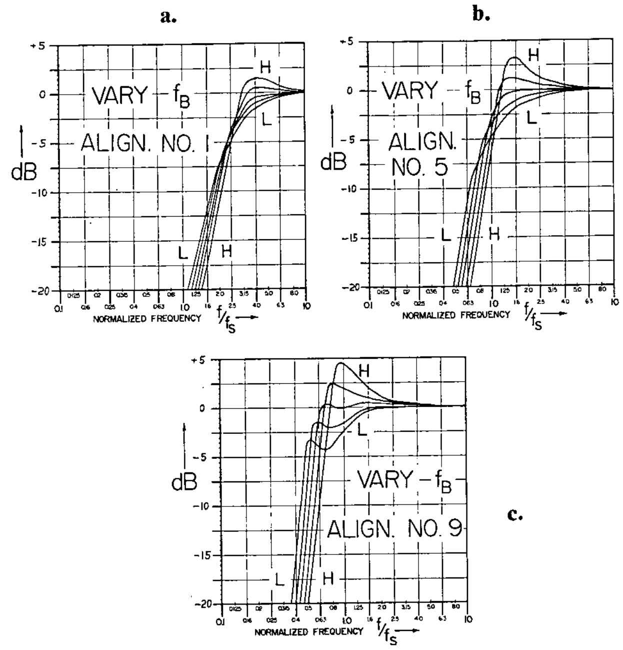


Fig. 7. Effect of variation of box resonance frequency f_B on alignments. **a.** No. 1. **b.** No. 5. **c.** No. 9. Step factors of 0.794, 0.891, 1.000, 1.122, and 1.259 are shown. Thiele system parameters f_S, Q_T, V_B, and V_{AS} are held constant. Note increasing sensitivity as alignment number gets higher.

$$S_x^M(w) = \frac{dM(w)/M(w)}{dx/x} = \frac{x}{M(w)}\frac{\partial M(w)}{\partial x} \approx \frac{\Delta M(w) \text{ in } \%}{\Delta x \text{ in } \%}. \quad (11)$$

Notice that S_x^M is a normalized variable which indicates the relationship between percentage shifts in $M(W)$ and x. The concept of sensitivity is theoretically valid only for infinitesimal shifts, but is accurate enough for engineering purposes for shifts up to about 15% in the independent parameter. For illustrative purposes, a sensitivity value of +1 would indicate a 5% increase in x and would reflect in an approximate 5% increase in M (5% is about 0.4 dB).

SENSITIVITY OF THIELE SYSTEM PARAMETERS

To compute the required partial derivatives, Eq. (2) is first written as a magnitude function of $\omega_N = \omega/\omega_S$:

$$E = E(\omega_N) = |E(j\omega_N)| = \frac{\omega_N^4}{\{[\omega_N^4-(1+h^2+\alpha)\omega_N^2+h^2]^2+(\omega_N^2/Q_T^2)(h^2-\omega_N^2)^2\}^{1/2}}. \quad (12)$$

After much manipulation and pencil work, the sensitivity functions appear as follows.

Sensitivity of Driver Q

$$S_{Q_T}^E = \frac{Q_T}{E}\frac{\partial E}{\partial Q_T} = \frac{E^2(\omega_N^2-h^2)^2}{Q_T\,\omega_N^6}. \quad (13)$$

Sensitivity of Cabinet Tuning

$$S_h^E = \frac{h}{E}\frac{\partial E}{\partial h} = \frac{2h^2E^2}{\omega_N^8}\left\{\frac{\omega_N^2}{Q_T^2}(\omega_N^2-h^2) - [\omega_N^4-(1+h^2+\alpha)\omega_N^2+h^2](1-\omega_N^2)\right\}. \quad (14)$$

Sensitivity of Compliance Ratio

$$S_\alpha^E = \frac{\alpha}{E}\frac{\partial E}{\partial \alpha} = \frac{\alpha E_G^2}{\omega_N^6}[\omega_N^4-(1+h^2+\alpha)\omega_N^2+h^2]. \quad (15)$$

It can also be shown that

$$S_{V_{AS}}^E = S_\alpha^E = -S_{V_B}^E.$$

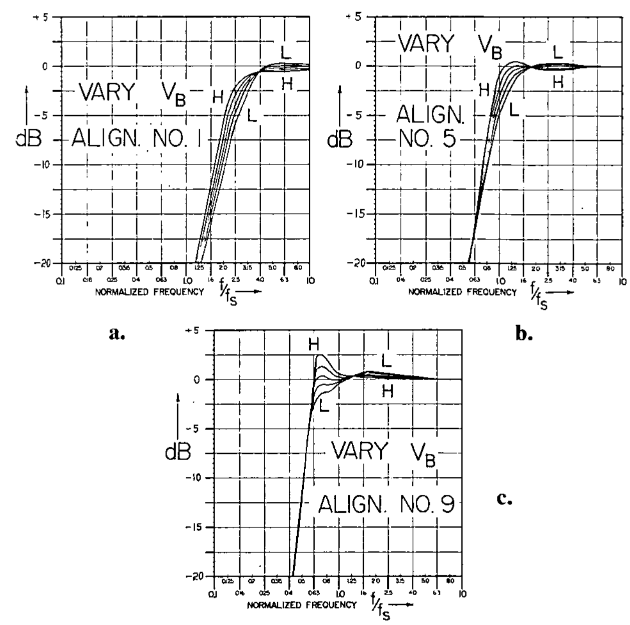


Fig. 8. Varation of box volume V_B (proportional to $1/\alpha$) on alignments. **a.** No. 1. **b.** No. 5. **c.** No. 9. Step factors of 0.794, 0.891, 1.000, 1.122, and 1.259 larger and smaller than the correct value are shown. Thiele system parameters f_S, f_B, Q_T, and V_{AS} are held constant.

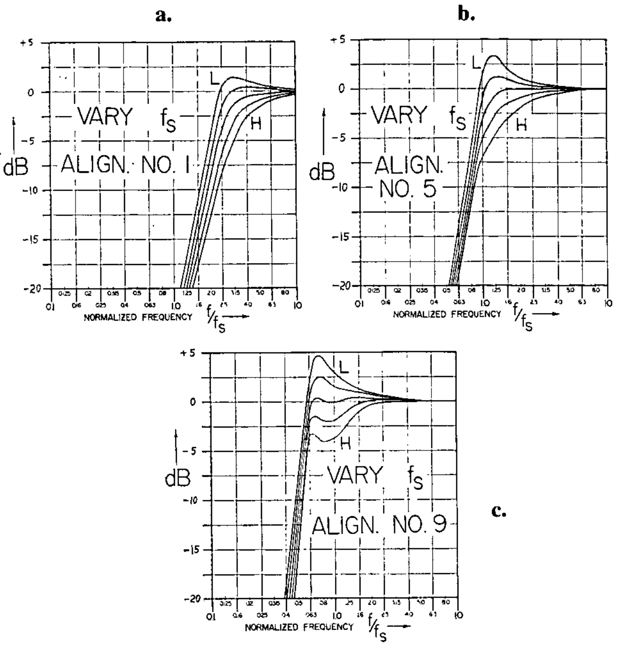


Fig. 9. Variation of driver free-air resonance frequency f_S on alignments. **a.** No. 1. **b.** No. 5. **c.** No. 9. Step factors of 0.794, 0.891, 1.000, 1.122, and 1.259 are illustrated. Thiele system parameters f_B, Q_T, V_B, and V_{AS} are held constant. Frequency scale is normalized to the correct value of f_S.

Sensitivity of Driver Resonance

$$S_{\omega_S}{}^E = \frac{\omega_S}{E}\frac{\partial E}{\partial \omega_S} =$$
$$\frac{E^2}{\omega_N{}^6}\left\{2[\omega_N{}^4-(1+h^2+a)\omega_N{}^2+h^2][\omega_N{}^2(1+a)-h^2]-\frac{\omega_N{}^2}{Q_T}(\omega_N{}^2-h^2)\right\}. \quad (16)$$

For this sensitivity, the normalized frequency variable ω_N is assumed to be normalized to the correct value ω_S of the alignment.

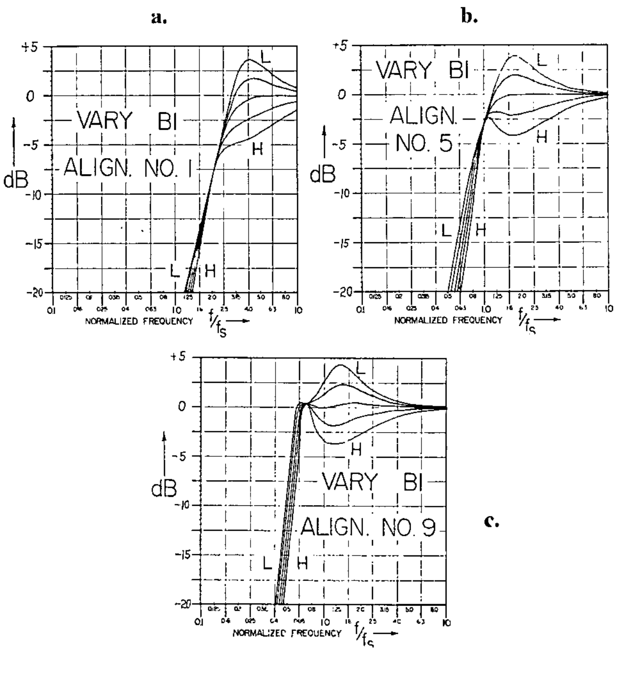


Fig. 10. Variation of driver Bl product on alignments. **a.** No. 1. **b.** No. 5. **c.** No. 9. Step factors of 0.794, 0.891, 1.000, 1.122, and 1.259 are shown. Driver physical parameters M_{MS}, C_{MS}, R_E and box parameters f_B, V_B were held constant.

SENSITIVITY OF FUNDAMENTAL DRIVER PARAMETERS

In like manner, the magnitude of Eq. (1) was derived with substitutions for ω_S, Q_T, and α made according to Eqs. (8), (9), and (10), yielding

$$E = E(\omega) = |E(j\omega)| =$$
$$\frac{\omega^4}{\left\{\left[\omega^4-\left(\omega_B{}^2+\frac{1}{M_{MS}C_{MS}}+\frac{S_D{}^2}{C_{AB}M_{MS}}\right)\omega^2+\frac{\omega_B{}^2}{M_{MS}C_{MS}}\right]^2+\frac{(Bl)^4\omega^2}{R_E{}^2M_{MS}{}^2}(\omega_B{}^2-\omega^2)^2\right\}^{1/2}}. \quad (17)$$

Neglecting air load masses on the driver cone, the required partial derivatives were calculated and the following sensitivity functions formed.

Sensitivity of Electromagnetic Coupling

$$S_{Bl}{}^E = \frac{(Bl)}{E}\frac{\partial E}{\partial (Bl)} = -\frac{2E^2(Bl)^4}{\omega^6R_E{}^2M_{MS}{}^2}(\omega_B{}^2-\omega^2)^2. \quad (18)$$

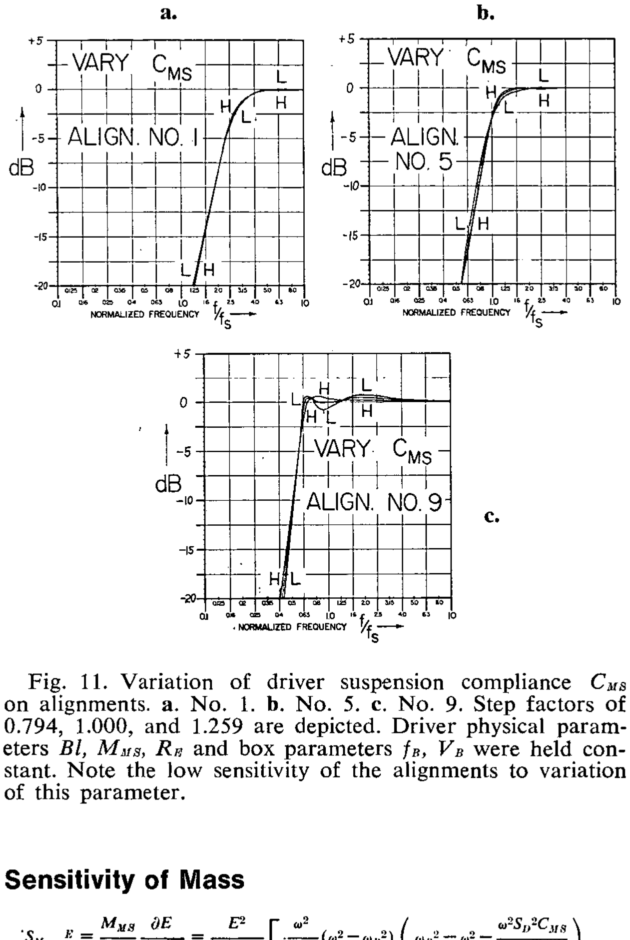


Fig. 11. Variation of driver suspension compliance C_{MS} on alignments. **a.** No. 1. **b.** No. 5. **c.** No. 9. Step factors of 0.794, 1.000, and 1.259 are depicted. Driver physical parameters Bl, M_{MS}, R_E and box parameters f_B, V_B were held constant. Note the low sensitivity of the alignments to variation of this parameter.

Sensitivity of Mass

$$S_{M_{MS}}{}^E = \frac{M_{MS}}{E}\frac{\partial E}{\partial M_{MS}} = \frac{E^2}{\omega^8M_{MS}}\left[\frac{\omega^2}{C_{MS}}(\omega^2-\omega_B{}^2)\left(\omega_B{}^2-\omega^2-\frac{\omega^2S_D{}^2C_{MS}}{C_{AB}}\right)\right.$$
$$\left.+\left(\omega_B{}^2-\omega^2-\frac{\omega^2S_D{}^2C_{MS}}{C_{AB}}\right)^2\frac{1}{C_{MS}{}^2M_{MS}}+\frac{(Bl)^4\omega^2(\omega_B{}^2-\omega^2)^2}{R_E{}^2M_{MS}}\right]. \quad (19)$$

Sensitivity of Suspension Compliance

$$S_{C_{MS}}{}^E = \frac{C_{MS}}{E}\frac{\partial E}{\partial C_{MS}} =$$
$$\frac{E^2}{\omega^8C_{MS}}\left[\omega^4-\left(\omega_B{}^2-\frac{S_D{}^2}{C_{AB}M_{MS}}-\frac{1}{C_{MS}M_{MS}}\right)\omega^2+\frac{\omega_B{}^2}{C_{MS}M_{MS}}\right](\omega_B{}^2-\omega^2). \quad (20)$$

The computer was used to evaluate these sensitivity functions by using Eq. (1) or (2) and working directly from the definition of the sensitivity function, assuming

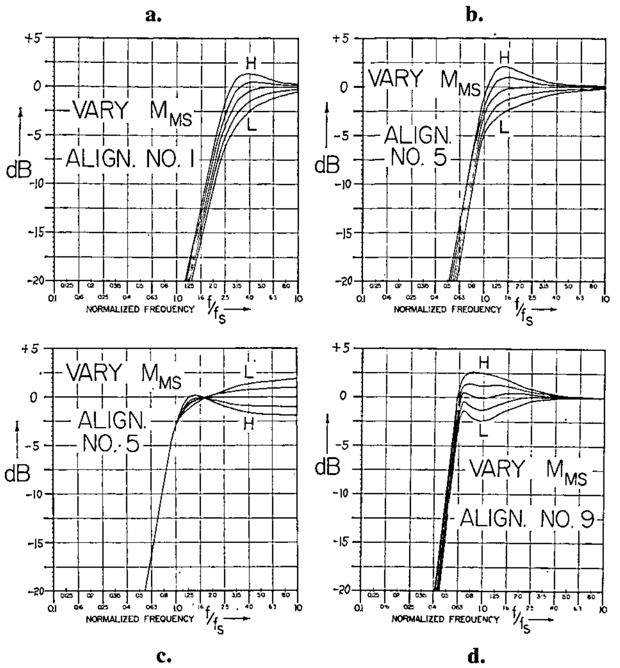


Fig. 12. Variation of driver effective moving mass M_{MS} on alignments. **a.** No. 1. **b**, **c**. No. 5. **d**. No. 9. Step factors of 0.794, 0.891, 1.000, 1.122, and 1.259 are shown. Driver physical parameters Bl, C_{MS}, R_E and box parameters f_B, V_B were held constant. **c** has been unnormalized in the passband to show absolute response variations because of shifting driver efficiency.

a 0.1% change in the independent parameter. The computer output for the sensitivity functions of alignments 1, 5, and 9 is shown in Figs. 13 and 14.

OBSERVATIONS ON VARIATION OF SYSTEM PARAMETERS

Changes in Driver Q

Examination of the graphical data (Figs. 6 and 13**a**) and Eq. (13) shows that the magnitude response is effected by variations of Q_T mostly at frequencies about an octave above and below the box resonance frequency. The maximum sensitivities occur at the frequencies f_L and f_H (the frequencies at which the input impedance is maximum for the speaker mounted in the vented box as defined by Thiele). The sensitivity functions indicate that all the alignments are equally sensitive (peaks of +1.0 in sensitivity for all the alignments). Also shown is the independence of the response with respect to Q_T at the box resonance frequency (if the response is normalized in the passband for ω large). These results are in agreement with [1, Eqs. (42) and (43)].

Changes in Cabinet Volume

Increases in box volume reflect an increased output at frequencies at or near the box resonance frequency (see Figs. 8 and 13**b**). The single maximum of about +1.0 in sensitivity occurs at the box resonance frequency. Above f_H and below f_L increases in box volume actually cause a slight decrease in system output. The response at the frequencies f_L and f_H is volume independent for small shifts in volume. All the alignments are about equally sensitive to shifts in the volume ratio. The sensitivity decays to zero for large and small frequencies (same behavior as sensitivity to Q_T).

Changes in Helmholtz Frequency

An increase in the box resonance frequency causes an increase in the output immediately above the optimum required box resonance frequency and a decrease in the output below this frequency (Figs. 7, 13**c**, and 13**d**). For extremely high frequencies the sensitivity decays to zero. For very low frequencies the sensitivity approaches −2 (this behavior at low frequencies is expected because the denominator of Eq. (1) approaches $\omega_S{}^2\omega_B{}^2$). The higher numbered alignments exhibit an extreme sensitivity to shifts of ω_B at frequencies near the optimum box frequency. Alignment 9 exhibits a sensitivity peak of nearly −5 just below the optimum box frequency.

Changes in Driver Resonance

Examination of Figs. 9, 13**e** and 13**f** shows how the response shifts if the speaker free-air resonance frequency, f_S, is changed. In a practical sense, independent shifts in f_S without shifts in Q_T and V_{AS} are most unlikely (for example, a shift in suspension compliance would change all three). A shift in f_S only implies some fancy juggling of the driver fundamental physicial parameters (see Eqs. (8), (9), and (10)). The sensitivity and variation of f_S were included in this study mainly for completeness.

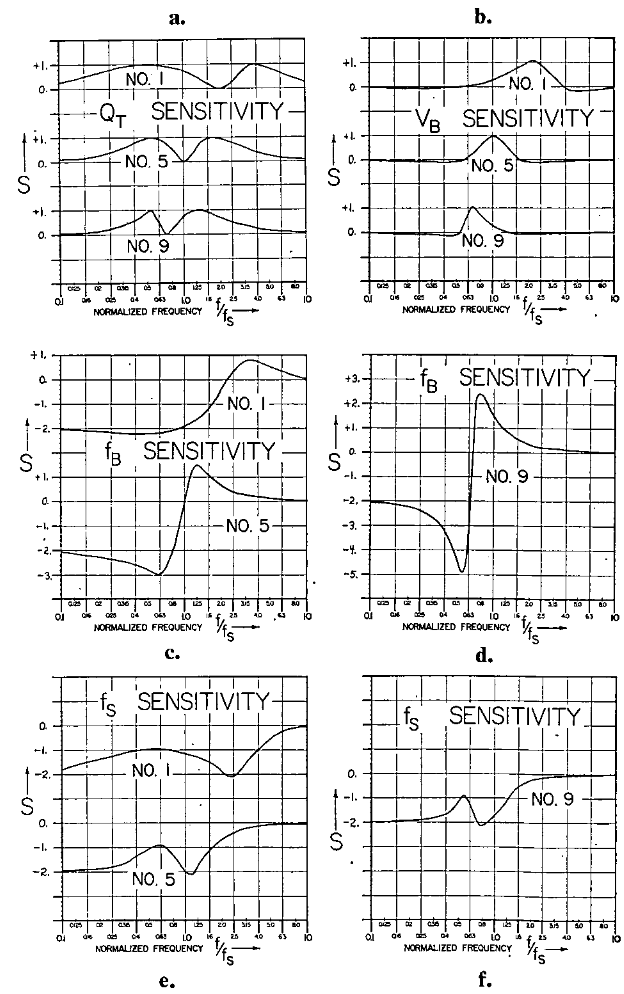


Fig. 13. Sensitivity functions for alignment nos. 1, 5, and 9 for variations of Thiele system parameters. **a.** Q_T. **b.** V_B (or $1/\alpha$). **c**, **d**. f_B. **e**, **f**. f_S. Note the extreme values of the sensitivity for shifts of f_B on alignment 9 (**d**).

The graphs indicate a decrease in the response for positive shifts in ω_S. A negative peak of -2.2 in sensitivity is found approximately at the alignment box resonance frequency. A negative dip of -1.0 is observed at a frequency of about two thirds of an octave below the speaker resonance frequency. The sensitivity approaches -2 for low frequencies and 0 for high frequencies. Equal sensitivities for all the alignments are exhibited by variation of ω_S.

Changes in Electromagnetic Coupling

Positive shifts in the driver Bl product cause decreases in the alignment response (Figs. 10 and 14**a**) in the frequency range about an octave above and below the box resonance frequency f_B (maximum sensitivity of -2 at f_L and f_H). Zero sensitivity is exhibited at f_B. Because of the singular relationship between Q_T and Bl shown in Eq. (9), the variations and sensitivities exhibited by these parameters are very similar. The sensitivity of Bl is just twice the negative (Bl appears squared in the denominator) of the Q_T sensitivity. Roughly equal Bl sensitivities are exhibited by all the fourth-order alignments.

Changes in Mass

The variational responses (Figs. 12 and 14**b**) show that the alignment output increases for increases in driver moving mass (relative to the normalized upper pass-band level). Fig. 12**c** shows the variational effect of M_{MS} on alignment 5 but with the pass-band level unnormalized to reflect shifts in driver efficiency. The sensitivity functions (Fig. 14**b**) exhibit a uniform level of roughly $+1$ for all frequencies less than about an octave above box resonance frequency f_B. Equal sensitivities to shifts in driver mass are shown by all the fourth-order alignments.

Changes in Compliance

Examination of Figs. 11 and 14**c** show a surprisingly low sensitivity of alignment output with respect to shifts in the driver suspension compliance. This low sensitivity seems intuitively correct for the quasi-Butterworth [1, p. 389] alignments 1 to 4 because the total system stiffness is predominantly that of the box ($0.095 \leq 1/\alpha \leq 0.34$). However, the low sensitivity is not quite so evident for the Butterworth and Chebyshev alignments 5 to 9 ($0.707 \leq 1/\alpha \leq 2.6$).

A rather elegant explanation of this phenomenon is as follows.[4] A shift in suspension compliance above or below the correct value for a driver designed for a specific Thiele vented-box alignment (i.e., specific box volume V_B, frequency f_B, and cutoff frequency f_3) changes the Thiele parameters (f_S, V_{AS}, and Q_{TS}) in such a complementary manner as to realign the driver to fit quite closely a higher or lower numbered alignment in the same box (same V_B, f_B, and approximately the same f_3). For example, if a driver designed for alignment 6 with a given box increases in compliance by a factor of 1.34, the new Thiele parameters for the driver and system are found to be very close to those of alignment 5. In general the new alignment obtained as the result of a compliance shift is not limited to the discrete-numbered alignments of the table, but may fall at any intermediate point. The important fact is the response curve shape and values of f_3 and f_B change only slightly.

[4] This was pointed out to the author by one of his colleagues, Raymond J. Newman of Electro-Voice, Inc.

The low sensitivity of the Thiele alignments to shifts in driver suspension compliance is indeed quite fortunate, because it is this parameter which normally undergoes the widest variation in driver manufacturing and also is the most likely to shift with age and temperature.

CONCLUSIONS ON SYSTEM ALIGNMENT CONSIDERING SENSITIVITY FUNCTIONS

Thiele System Parameters

The graphs displaying sensitivity show that the system response is relatively insensitive to variations in Q_T and V_B (absolute sensitivities of about 1 or less), moderately sensitive to variations in f_S (absolute sensitivities of about 2 or less), and quite sensitive to variations of f_B for the higher numbered alignments (absolute sensitivities of less than about 5). These data indicate that for a specific alignment the box should be tuned quite accurately to the computed design frequency so that frequency response deviations are minimized.

Driver Physical Parameters

Observation of the sensitivity graphs for these parameters indicates that the vented-box output is quite insensitive to shifts in C_{MS} (absolute sensitivities of about 0.4 or less), somewhat sensitive to changes in M_{MS} (absolute sensitivities of less than 1.3), and moderately sensitive to variations of the Bl product (absolute sensitivities of 2 or less).

Computation of the sensitivity functions for the Thiele driver parameters with respect to the drivers fundamental physical parameters from Eqs. (8), (9), (10), and (11) yields

$$S_{M_{MS}}^{\omega_S} = S_{C_{MS}}^{\omega_S} = S_{C_{MS}}^{Q_T} = -\tfrac{1}{2} \tag{21}$$

$$S_{M_{MS}}^{Q_T} = \tfrac{1}{2} \tag{22}$$

$$S_{C_{MS}}^{\alpha} = 1 \tag{23}$$

$$S_{Bl}^{Q_T} = -2. \tag{24}$$

It has been the author's experience that out of a typical batch of two to four drivers of the same make and model, a variation of 10–20% in the Thiele parameters (f_S, V_{AS}, Q_{TS}) is not unusual. These Thiele parameter variations most likely occur because of significant changes in the driver suspension compliance (for example, a positive shift in C_{MS} of $+10\%$ will change V_{AS} by $+10\%$, ω_S by -5%, and Q_{TS} by -5%), coupled with minor changes in the other driver physical parameters. This type of assumed parameter change will *not* affect the output of the vented-box system to any great extent because of the low sensitivity of $S_{C_{MS}}^{E}$ (Figs. 11 and 14**c**) and the alignment hopping effect described in a previous section. If the Thiele parameter variations are caused by

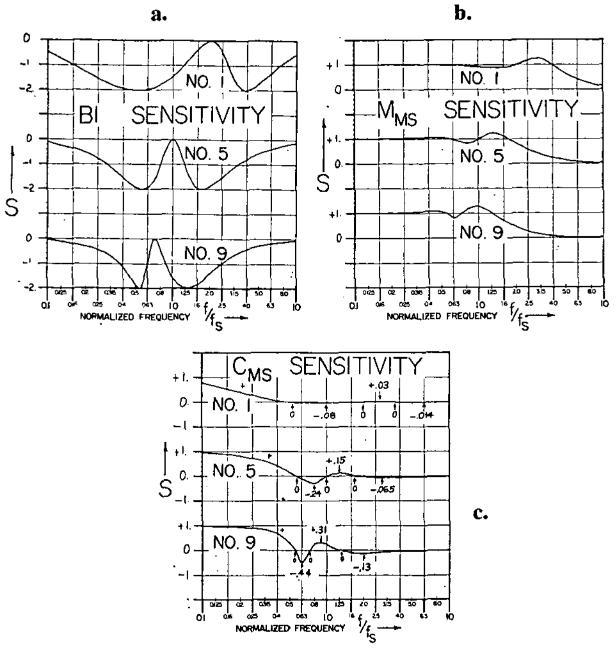


Fig. 14. Sensitivity functions for alignment nos. 1, 5, and 9 for variations of driver physical parameters. **a** B*l*. **b** M_{MS}. **c** C_{MS}. Note the small sensitivity for shifts in the driver suspension compliance C_{MS} (**c**).

significant shifts in all the driver physical parameters, the vented-box system designer has no recourse except to individually tune each box to the particular driver it will be used with.

CONCLUSION

The computer-run frequency response curves shown for all the Thiele alignments help the designer of vented-box loudspeaker systems to make intelligent selections of response shapes which will suit his requirements.

The derived sensitivity functions and parameter variation graphs demonstrate how the vented-box output changes for shifts in particular system and driver parameters. The sensitivity functions derived for variations of the box resonance frequency for the vented loudspeaker cabinet show the extreme sensitivity of the system output to variations of this parameter for the Chebyshev alignments. The vented-box response is shown to be quite insensitive to changes in the driver suspension compliance.

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Note: Mr. Keele's biography appeared in the January/February issue of the Journal.