AWASP: AN ACOUSTIC WAVE ANALYSIS AND SIMULATION PROGRAM

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AWASP: AN ACOUSTIC WAVE ANALYSIS AND SIMULATION PROGRAM

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A report on the preliminary version of a general purpose computer program for the analysis and simulation of two-dimensional acoustic spaces under steady-state sinusoidal conditions. The program employs finite element methods to predict the sound pressure magnitude and phase in an arbitrary region at any desired frequency. AWASP is similar in concept and usage to well known electronic circuit analysis programs such as ECAP and SPICE.

The program has wide application in the analysis and design of a large number of electro-acoustic situations including: sound in ducts, horn propagation, sound in enclosures, diffraction effects, etc. Applications analyzed include: sound in straight and bent pipes, sound in a live room, and simulation of a Helmholtz resonator.

INTRODUCTION

The finite-element method [1] which originally started as a process of structural analysis has become recognized as a general method of wide applicability to Engineering and physical science problems [2] [3] This paper describes an application of this method in development of a computer program for two-dimensional (2D) acoustic modeling.

The goal of the program was to essentially solve the two-dimensional steady-state wave equation in an arbitrary region (with arbitrary boundaries and sources) and yield a detailed plot of the sound pressure magnitude and phase at a finite number of points in the region. The input and output data of the program are in a form which allows rapid problem definition and result interpretation. The name chosen for the program is "AWASP" which stands for Acoustic Wave Analysis and Simulation Program.

PROGRAM DESCRIPTION

AWASP is a general-purpose computer program employing finite element technology for analysis and simulation of two-dimensional acoustic spaces under steady-state sinusoidal conditions. Spaces can be of arbitrary size and shape and contain and/or be bounded by arbitrary complex (having both magnitude and phase) boundaries. Absorption coefficients at boundaries including anechoic terminations can be easily entered. Any number of independent pressure sources having complex source strengths and source impedances can be simulated.

FINITE ELEMENT METHOD

"The finite element method," quoting Zienkiewicz [1, p. vii], "is essentially a process through which a continuum with infinite degrees of freedom can be approximated by an assemblage of subregions (or elements) each with a specified but now finite number of unknowns. Further, each such element interconnects with others in a way familiar to engineers dealing with discrete structural or electrical assemblies." The resultant assemblage of subregions or elements which are all interconnected at a discrete number of nodal points is then analyzed using formalized matrix techniques to yield output data at each node.

The finite element analysis method yields a large set of matrix simultaneous equations (one equation and one unknown or more for each node) which are then solved using the digital computer. To a large degree the method is essentially a brute force analysis technique which has become feasible in the last decade because of rapidly increasing computer power and the decreasing cost of computation. It can be shown that the method converges to the exact solution in any region as the number of elements is increased.

AVAILABLE PROGRAMS

A number of large computer programs for structural analysis which use the finite element method exist; a partial list includes: ANSYS, NASTRAN, SUPERB, etc. [4], [5], [6]. Some of these programs even have the capability to solve wave equation type problems by virture of having a 2D or 3D wave element [4]. Unfortunately, due to the extreme generality and very large size of these programs (ANSYS code takes about 2 megabytes) running them for small to medium size acoustic problems is quite complicated and expensive. In addition, the input/output is often in a form which is clumsy, hard to set up and interpret.

The intent here was to develop a finite element computer program specifically tailored for analyzing acoustic problems which would run on a small to medium size computer efficiently.

PROGRAM INPUT/OUTPUT

A graphical input/output format was chosen which could be used with any alphanumeric terminal or printer. Two-dimensional acoustic boundary regions are input to the computer in a form which uses images of the space with symbols having specific meanings. The output of the program is printed on the same image with separate sound level magnitude and phase plots.

To simplify the mathematical formulation and the resultant program a square finite element with 4 corner data nodes was chosen. The region to be analyzed is set up using these square 2D elements as basic building blocks. In the next section, examples of actual program input and output for several cases will clarify these statements and illustrate the versatility of the program.

AWASP has the capability of modeling a number of different types of boundaries including complex and anechoic conditions. Any number of pressure or velocity sourcescan be simulated with arbitrary source impedances. Refer to the appendix for a complete list and description of the programs input options.

APPLICATION

Four relatively simple test cases were analyzed with AWASP to illustrate its use:
Case 1. Square reverberent room,
Case 2. Straight pipe with matched source and load,
Case 3. Bent pipe with matched source and load, and
Case 4. Helmholtz resonator with duct excitation.

In every case a series of pressure magnitude and phase plots were run at several frequencies to show the changing distributions with frequency. Experience with using the program on a large number of cases reveals that 6 to 8 elements per wavelength are required at the highest analysis frequency for sufficient accuracy. The speed of propagation in each plot is 1 element per second. Cited frequencies are in wavelengths per element (wpe).

Case 1. 4 x 4 Element Square Reverberation Room.

The acoustic space simulated is a square room with perfectly reflective walls with a single source in the corner. Fig. 1 shows the format for the input image for this situation.

The 16 finite elements are indicated by X's.

1. Each finite element (X's in Fig. 1) has 4 data nodes associated with it.

2. The data nodes are indicated by +'s and Ø's.

3. The +'s indicate a node without an associated source or load while the Ø's indicate a perfectly reflective boundary (absorption coefficient of zero). Note that these two symbols can be used interchangeable on a boundary because a "+" node without an associated element "X" is the same as a "Ø" node.

4. The corner source is indicated by a "$" which specifies a unit pressure behind a unit specific acoustic resistance (pressure/velocity).

The program's pressure magnitude and phase plots are shown in Fig. 2 for several frequencies. Please refer to figure caption for descriptive information.

To illustrate the effect of increased resolution and accuracy afforded by using a larger number of elements, two additional runs were made at double (8 x 8 elements) and quadruple (16 x 16 elements) dimensions for the frequency corresponding to the first room node (Fig. 2e). These outputs are shown in Figs. 3 and 4 respectively.

Case 2. 2 x 12 Element Straight Pipe with Matched Source and Load.

The program simulates a one dimensional transmission line in two dimensions by modeling a plane wave source and load on the ends of a 2 x 12 element region. The source and load impedances are chosen to correctly match the characteristic impedance of the line (unit magnitudes and zero phase).

An exact analytical solution to this problem shows that the pressure magnitude should be exactly 0.5 (-6.0 dB) at all points in the pipe for all frequencies. The corresponding phase distribution should indicate an in-phase condition (0^{o}) at the source with a linear phase lag as one proceeds toward the load. The amount of lag or delay depends on the frequency. As an example, a line which is one-half wavelength long would have a phase of -180^{o} at the load with a phase of -90^{o} midway between source and load.

The following output plots give a good idea of the accuracy of the method for small number of elements per wavelength . The input image for this case is shown in Fig. 5 with the outputs for several frequencies shown in Fig. 6. The "#" symbol indicates a unit real load.

Case 3. 2 x 12 Element Bent Pipe with Matched Source and Load.

The program is used to model the pipe of Case 2 but with the last third of the pipe bent at right angles. The input image for this case is shown in Fig. 7 with the output at a number of frequencies displayed in Fig. 8.

For low frequencies (Fig. 8a-b) the SPL distribution is essentially the same as the straight pipe with the sound flowing smoothly around the bend. For higher frequencies (Fig. 8c-f) however, where the distance to be bend is one-tenth wavelength or more, the bend generates reflections which interfere with the sound transmission. Note the strong standing waves in the first part of the pipe in Fig. 8f when the source is one wavelength from the bend and the resultant loss of level at the load.

Case 4. Helmholtz Resonator with Duct Excitation.

A Helmholtz resonator is modeled in this case with a rather stiff source (low source resistance) driving the duct or vent. The simple resonator of Fig. 9 is depicted by a 3 x 3 element square cavity coupled to a 1 x 4 element pipe. Analysis reveals resonance should occur at roughly $f \approx .025$ or $N \approx 40$. Fig. 10 displays the output plots for several frequencies. Note the large buildup of pressure (+16 dB above source) that occurs at $f = .024$ or $N = 41.5$.

PROGRAM LIMITATIONS

The size of the space to be analyzed is limited essentially by the speed and memory availability of the host computer. The current time share version of the program which is written in BASIC language (UCS SUPER BASIC) does not use sophisticated memory management techniques to minimize high speed memory usage. The whole program and associated data must exist in high speed memory at the same time. Most of the computer memory and execution time is taken up in the simultaneous equation solution subroutine which uses a version of Gaussian elimination [7].

Analysis shows that approximately $2N^3$ memory locations for number storage are required for solution where N is the number of elements per side of analyzed space. For example a 10 x 10 element region require about 2 K and a 50 x 50 region about 250 K, etc.

Regions up to 20 x 20 = 400 elements or approximately 3 x 3 wavelengths (at roughly 6 elements per wavelength) can be analyzed using the current time share version of the program with 24 K words of memory available. Analysis reveals that regions of roughly 10 x 10 wavelengths or 60 x 60 = 3,600 elements are required to properly predict the polar pattern of a horn for instance. Please note, however, that regions as small as 2 x 2 = 4 up to 4 x 4 = 16 elements can in a number of cases yield very useful information (see section on examples of program input and output).

CONCLUSION

A brief description of the preliminary version of a computer program to solve generalized 2D acoustic spaces under steady-state sinusoidal conditions has been presented. The primary limitations of the program are for the most part based on the speed and memory capability of the host computer and secondarily on the programs methods of solution. Future enhancements to the program will include rewrite of the critical portions of the program to improve speed and efficiency and graphical output to aid in comprehension of the generated data.

REFERENCES

[1] O.C. Zienkiewicz, The Finite Element Method in Engineering Science, (McGraw-Hill, London 1971).

[2] C.J. Young and M.J. Crocker, "Prediction of Transmission Loss in Mufflers by the Finite-Element Method," J. Acoust. Soc. Am., vol. 57, pp 144-148 (Jan. 1975).

[3] K. Suzuki and I. Nomoto, "Computerized Analysis and Observation of the Vibration Modes of a Loudspeaker Cone," presented at the 58th convention of the Audio Eng. Soc., Preprint no. 1280(O-5) (Nov. 4-7, 1977, New York).

[4] ANSYS, Engineering Analysis System, Swanson Analysis Systems, Inc., Elizabeth, PENN. Available from UCS (United Computing Systems) and CDC (Control Data Corp. Cybernet Service).

[5] NASTRAN, NASA Structural Analysis Computer System, available from a number of computer service bureaus.

[6] SUPERB, A General Purpose Isoparametric Finite Element Program, Structural Dynamic Research Corp., available from CSC (Computer Science Corp., El Segundo, CA).

[7] W.S. Dorn and D.D. McCracken, Numerical Methods with Fortran IV Case Studies, (John Wiley & Sons, Inc., New York 1972).

APPENDIX

PROGRAM INPUT OPTIONS

Boundary and source/load conditions are input to the computer by use of different symbols at the node locations in the input graphical image. These symbols are:

+ = Node which has no source, load, or absorption associated with it. Indicates communication between elements only.

$ = Source node of unit pressure and zero phase with unit real source impedance.

A to Z = Source/Load node of arbitrary source strength and impedance. Both magnitude and phase can be independently specified for both strength and impedance. Constant pressure sources are modeled with low source impedances and constant velocity sources with high source impedances. Load only nodes are modeled by setting source strengths to zero.

= Load node with unit real impedance. Also specifies an absorption coefficient of unity.

Ø - 9, # = Absorption node with absorption coefficient α as follows.

Symbol	α	
Ø (or +)	0	(perfect reflector)
1	.1	
2	.2	
3	.3	
4	.4	
5	.5	
6	.6	
7	.7	
8	.8	
9	.9	
#	1.0	(perfect absorber)

```
REGION DIMENSIONS = 5 NODES DOWN AND 5 NODES ACROSS.

4X4 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      Ø3/16/78.
+++++++++++++++ UNCHECKED INPUT IMAGE +++++++++++++++
  Ø     Ø     Ø     Ø     Ø
     X     X     X     X
  Ø     +     +     +     Ø
     X     X     X     X
  Ø     +     +     +     Ø
     X     X     X     X
  Ø     +     +     +     Ø
     X     X     X     X
  $     Ø     Ø     Ø     Ø

+++++++++++++++++++++ SOURCE/LOAD LIST +++++++++++++++++++++
IDENT.  TYPE             SOURCE                IMPEDANCE
                 MAGNITUDE     PHASE     MAGNITUDE     PHASE
                               DEGS.                   DEGS.
  $     SOURCE     1.ØØØØØØ      .Ø        1.ØØØØØØ      .Ø


NUMBER OF FINITE ELEMENTS = 16
          NUMBER OF NODES = 25
           HALF BANDWIDTH = 6
```

a.

```
LIST

 Ø3/16/78. 1Ø.39.28.
 LIST OF  DATA

1Ø 5,5
2Ø "Ø Ø Ø Ø Ø"
3Ø " X X X X "
4Ø "Ø + + + Ø"
5Ø " X X X X "
6Ø "Ø + + + Ø"
7Ø " X X X X "
8Ø "Ø + + + Ø"
9Ø " X X X X "
95 "$ Ø Ø Ø Ø"
1ØØ Ø,Ø,Ø,Ø
*RDY*
```

b.

Fig. 1. Computer input required to describe a square 4 x 4 finite element reverberation room with a single corner source. The corner source has unit real pressure amplitude and unit specific acoustic resistance (pressure/velocity). Perfectly reflective walls are indicated by zeros ("Ø") along the boundaries which specify an absorption coefficient by zero.

a. Computer printed input image.
b. Raw data as listed in input file.

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 100000
                          FREQUENCY (1/N) = .1E-04
                          OMEGA (2*PIE/N) = .62831853E-04

4X4 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      03/16/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                 LEVEL IN DB
  -.0   -.0   -.0   -.0   -.0
     X     X     X     X
  -.0   -.0   -.0   -.0   -.0
     X     X     X     X
  -.0   -.0   -.0   -.0   -.0
     X     X     X     X
  -.0   -.0   -.0   -.0   -.0
     X     X     X     X
  -.0   -.0   -.0   -.0   -.0
```

AMPLITUDE dB

λ/25,000

SOURCE

```
********** QUICK PRINT PHASE DISTRIBUTION **************
                PHASE IN DEGREES
   0     0     0     0     0
     X     X     X     X
   0     0     0     0     0
     X     X     X     X
   0     0     0     0     0
     X     X     X     X
   0     0     0     0     0
     X     X     X     X
   0     0     0     0     0
```

PHASE

DEGS.

a. $f = \frac{1}{100{,}000}$ wavelengths per element (wpe)

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 512
                          FREQUENCY (1/N) = .1953125E-02
                          OMEGA (2*PIE/N) = .12271846E-01

4X4 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      03/16/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                 LEVEL IN DB
  -.6   -.6   -.6   -.6   -.6
     X     X     X     X
  -.6   -.6   -.6   -.6   -.6
     X     X     X     X
  -.6   -.6   -.6   -.6   -.6
     X     X     X     X
  -.6   -.6   -.6   -.6   -.6
     X     X     X     X
  -.6   -.6   -.6   -.6   -.6
```

λ/128

```
********** QUICK PRINT PHASE DISTRIBUTION **************
                PHASE IN DEGREES
  -22   -22   -22   -22   -22
     X     X     X     X
  -22   -22   -22   -22   -22
     X     X     X     X
  -22   -22   -22   -22   -22
     X     X     X     X
  -22   -22   -22   -22   -22
     X     X     X     X
  -22   -22   -22   -22   -22
```

b. $f = \frac{1}{512}$ wpe

Fig. 2. Computer output at several frequencies for the square reverberation room of Fig. 1. At each frequency a pressure magnitude (upper plot) and phase (lower plot) distribution is displayed. The distributions cover the frequency range from (a.) very low to (i) the frequency of the first room resonance mode where the sides of the room are one-half wavelength long.

a. Shows the distribution at the very low frequency of $f = 1 \times 10^{-5}$ wavelengths per element (wps) which is about 14 octaves below the first room mode. Note the constant pressure amplitude and phase distributions with the respective values being equal to the source (0 dB and 0^{o}).

b.- f. As the frequency increases the pressure amplitude decreases and the phase shows an ever increasing lag.

g.- h. A diagonal node line appears close to the corner source and shifts toward the center diagonal of the room as the frequency increases.

i. This is the frequency of the first room resonance mode. Note the amplitude node or null line extending diagonally from corner to corner with divided regions being 180^{o} out of phase.

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 256
                              FREQUENCY (1/N) = .390625E-02
                              OMEGA (2*PIE/N) = .24543693E-01

4X4 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      03/16/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                     LEVEL IN DB
 -2.0  -2.0  -2.0  -2.0  -2.0
    X     X     X     X
 -2.0  -2.0  -2.0  -2.0  -2.0
    X     X     X     X
 -2.0  -2.0  -2.0  -2.0  -2.0     λ/64
    X     X     X     X
 -2.1  -2.0  -2.0  -2.0  -2.0
    X     X     X     X
 -2.1  -2.1  -2.0  -2.0  -2.0

********** QUICK PRINT PHASE DISTRIBUTION **************
                   PHASE IN DEGREES
 -39   -39   -39   -39   -39
    X     X     X     X
 -39   -39   -39   -39   -39
    X     X     X     X
 -39   -39   -39   -39   -39
    X     X     X     X
 -39   -39   -39   -39   -39
    X     X     X     X
 -39   -39   -39   -39   -39
```

c. $f = \frac{1}{256}$ wpe

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 128
                              FREQUENCY (1/N) = .78125E-02
                              OMEGA (2*PIE/N) = .49087385E-01

4X4 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      03/16/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                     LEVEL IN DB
 -5.2  -5.2  -5.2  -5.2  -5.2
    X     X     X     X
 -5.2  -5.2  -5.2  -5.2  -5.2
    X     X     X     X
 -5.3  -5.3  -5.2  -5.2  -5.2     λ/32
    X     X     X     X
 -5.4  -5.3  -5.3  -5.2  -5.2
    X     X     X     X
 -5.8  -5.4  -5.3  -5.2  -5.2

********** QUICK PRINT PHASE DISTRIBUTION **************
                   PHASE IN DEGREES
 -59   -59   -59   -59   -59
    X     X     X     X
 -59   -59   -59   -59   -59
    X     X     X     X
 -59   -59   -59   -59   -59
    X     X     X     X
 -59   -59   -59   -59   -59
    X     X     X     X
 -59   -59   -59   -59   -59
```

d. $f = \frac{1}{128}$ wpe

Fig. 2 (continued)

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 64
                              FREQUENCY (1/N) = .15625E-01
                              OMEGA (2*PIE/N) = .9817477E-01

4X4 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      03/16/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                LEVEL IN DB
-10.0 -10.0 -10.0  -9.9  -9.9
    X     X     X     X
-10.1 -10.1 -10.0  -9.9  -9.9
    X     X     X     X
-10.4 -10.3 -10.1 -10.0 -10.0      λ/16
    X     X     X     X
-10.9 -10.6 -10.3 -10.1 -10.0
    X     X     X     X
-12.5 -10.9 -10.4 -10.1 -10.0

********** QUICK PRINT PHASE DISTRIBUTION **************
               PHASE IN DEGREES
 -76   -76   -76   -76   -76
    X     X     X     X
 -76   -76   -76   -76   -76
    X     X     X     X
 -76   -76   -76   -76   -76
    X     X     X     X
 -76   -76   -76   -76   -76
    X     X     X     X
 -76   -76   -76   -76   -76
```

e. $f = \frac{1}{64}$ wpe

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 32
                              FREQUENCY (1/N) = .3125E-01
                              OMEGA (2*PIE/N) = .19634954

4X4 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      03/16/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                LEVEL IN DB
-15.4 -15.3 -15.0 -14.8 -14.7
    X     X     X     X
-15.7 -15.5 -15.2 -14.9 -14.8
    X     X     X     X
-16.8 -16.3 -15.6 -15.2 -15.0      λ/8
    X     X     X     X
-19.7 -17.7 -16.3 -15.5 -15.3
    X     X     X     X
-45.1 -19.7 -16.8 -15.7 -15.4

********** QUICK PRINT PHASE DISTRIBUTION **************
               PHASE IN DEGREES
 -90   -90   -90   -90   -90
    X     X     X     X
 -90   -90   -90   -90   -90
    X     X     X     X
 -90   -90   -90   -90   -90
    X     X     X     X
 -90   -90   -90   -90   -90
    X     X     X     X
 -90   -90   -90   -90   -90
```

f. $f = \frac{1}{32}$ wpe

Fig. 2 (continued)

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 16
                         FREQUENCY (1/N) = .625E-01
                         OMEGA (2*PIE/N) = .39269908

4X4 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      03/16/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                 LEVEL IN DB
-19.9 -19.3 -18.1 -17.2 -16.9
    X     X     X     X
-21.2 -20.3 -18.8 -17.6 -17.2
    X     X     X     X
-28.1 -24.5 -20.8 -18.8 -18.1        λ/4
    X     X     X     X
-25.3 -51.9 -24.5 -20.3 -19.3
    X     X     X     X
-11.9 -25.3 -26.1 -21.2 -19.9

********** QUICK PRINT PHASE DISTRIBUTION **************
                PHASE IN DEGREES
-105  -105  -105  -105  -105
    X     X     X     X
-105  -105  -105  -105  -105
    X     X     X     X
-105  -105  -105  -105  -105
    X     X     X     X
  75  -105  -105  -105  -105
    X     X     X     X
  75    75  -105  -105  -105
```

g. $f = \frac{1}{16}$ wpe, a node line appears

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 12
                         FREQUENCY (1/N) = .83333333E-01
                         OMEGA (2*PIE/N) = .52359878

4X4 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      03/16/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                 LEVEL IN DB
-20.9 -19.6 -17.2 -15.4 -14.8
    X     X     X     X
-24.0 -21.8 -18.4 -16.2 -15.4
    X     X     X     X
-34.8 -36.2 -22.8 -18.4 -17.2        λ/3
    X     X     X     X
-15.8 -22.1 -36.2 -21.8 -19.6
    X     X     X     X
 -7.4 -15.8 -34.8 -24.0 -20.9

********** QUICK PRINT PHASE DISTRIBUTION **************
                PHASE IN DEGREES
-115  -115  -115  -115  -115
    X     X     X     X
-115  -115  -115  -115  -115
    X     X     X     X
  65  -115  -115  -115  -115
    X     X     X     X
  65    65  -115  -115  -115
    X     X     X     X
  65    65    65  -115  -115
```

h. $f = \frac{1}{12}$ wpe

Fig. 2 (continued)

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 8
                              FREQUENCY (1/N) = .125
                              OMEGA (2*PIE/N) = .78539816

4X4 ELEMENT REVERB ROOM SINGLE CORNER SOURCE       03/16/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                  LEVEL IN DB
-25.1 -18.1  -5.0    .0   1.5
    X     X     X     X
-13.2 -27.4  -8.2  -1.7    .0
    X     X     X     X
 -4.5  -7.2 -30.6  -8.2  -5.0     λ/2
    X     X     X     X
  -.6  -1.8  -7.2 -27.4 -18.1
    X     X     X     X
  -.6    -.6  -4.5 -13.2 -25.1
SOURCE

********** QUICK PRINT PHASE DISTRIBUTION **************
                PHASE IN DEGREES
 -21   159   159   159   159
    X     X     X     X
 -21   -21   159   159   159
    X     X     X     X
 -21   -21   -21   159   159
    X     X     X     X
 -21   -21   -21   -21   159
    X     X     X     X
 -21   -21   -21   -21   -21
```

i. $f = \frac{1}{8}$ wpe, first room resonance mode

Fig. 2 (continued)

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 16
                          FREQUENCY (1/N) = .625E-01
                          OMEGA (2*PIE/N) = .39269908

8X8 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      02/27/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                     LEVEL IN DB
-37.9 -31.1 -16.9 -10.0  -5.7  -2.8   -.9    .1    .5
   X     X     X     X     X     X     X     X
-25.6 -38.6 -19.7 -11.2  -6.4  -3.3  -1.3   -.2    .1
   X     X     X     X     X     X     X     X
-15.7 -18.1 -40.5 -15.8  -8.7  -4.9  -2.6  -1.3   -.9
   X     X     X     X     X     X     X     X
 -9.6 -10.7 -15.0 -42.9 -14.2  -8.0  -4.9  -3.3  -2.8
   X     X     X     X     X     X     X     X
 -5.5  -6.2  -8.5 -13.7 -44.1 -14.2  -8.7  -6.4  -5.7
   X     X     X     X     X     X     X     X
 -2.8  -3.3  -4.8  -7.8 -13.7 -42.9 -15.8 -11.2 -10.0
   X     X     X     X     X     X     X     X
 -1.1  -1.4  -2.6  -4.8  -8.5 -15.0 -40.5 -19.7 -16.9
   X     X     X     X     X     X     X     X
  -.2   -.4  -1.4  -3.3  -6.2 -10.7 -18.1 -38.6 -31.1
   X     X     X     X     X     X     X     X
  -.1   -.2  -1.1  -2.8  -5.5  -9.6 -15.7 -25.6 -37.9

********** QUICK PRINT PHASE DISTRIBUTION **************
                  PHASE IN DEGREES
-10   170   170   170   170   170   170   170   170
   X     X     X     X     X     X     X     X
-10   -10   170   170   170   170   170   170   170
   X     X     X     X     X     X     X     X
-10   -10   -10   170   170   170   170   170   170
   X     X     X     X     X     X     X     X
-10   -10   -10   -10   170   170   170   170   170
   X     X     X     X     X     X     X     X
-10   -10   -10   -10   -10   170   170   170   170
   X     X     X     X     X     X     X     X
-10   -10   -10   -10   -10   -10   170   170   170
   X     X     X     X     X     X     X     X
-10   -10   -10   -10   -10   -10   -10   170   170
   X     X     X     X     X     X     X     X
-10   -10   -10   -10   -10   -10   -10   -10   170
   X     X     X     X     X     X     X     X
-10   -10   -10   -10   -10   -10   -10   -10   -10
```

SOURCE

$\lambda/2$

AMPLITUDE

dB

PHASE

DEGS.

Fig. 3. Double size 8 x 8 element amplitude/phase distribution for the reverberation room of Fig. 1 at the first room mode (2 times expansion of Fig. 2; with 4 times the elements).

AMPLITUDE

dB

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 32
                           FREQUENCY (1/N) = .3125E-01
                           OMEGA (2*PIE/N) = .19634954

16X16 ELEMENT REVERB ROOM SINGLE CORNER SOURCE      03/01/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                  LEVEL IN DB
-50.2 -43.4 -28.9 -21.6 -16.7 -13.0 -10.1  -7.8  -5.9  -4.3  -3.0  -2.0  -1.2   -.6   -.2    .1    .2
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
-37.8 -50.4 -31.7 -22.7 -17.3 -13.4 -10.4  -8.0  -6.0  -4.5  -3.2  -2.1  -1.3   -.7   -.3    .0    .1
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
-27.6 -29.9 -50.9 -27.1 -19.3 -14.6 -11.2  -8.6  -6.6  -4.9  -3.5  -2.5  -1.6  -1.0   -.5   -.3   -.2
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
-21.0 -22.1 -26.1 -51.7 -24.3 -17.2 -12.9  -9.8  -7.5  -5.6  -4.2  -3.0  -2.1  -1.4  -1.0   -.7   -.6
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
-16.4 -16.9 -18.9 -23.6 -52.8 -22.5 -15.7 -11.7  -8.9  -6.8  -5.1  -3.8  -2.8  -2.1  -1.6  -1.3  -1.2
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
-12.8 -13.2 -14.4 -16.9 -22.0 -54.0 -21.3 -14.8 -11.0  -8.4  -6.4  -5.0  -3.8  -3.0  -2.5  -2.1  -2.0
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
-10.0 -10.3 -11.1 -12.7 -15.5 -20.9 -55.3 -20.5 -14.3 -10.7  -8.2  -6.4  -5.1  -4.2  -3.5  -3.2  -3.0
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
 -7.7  -7.9  -8.6  -9.7 -11.6 -14.6 -20.2 -56.2 -20.2 -14.1 -10.7  -8.4  -6.8  -5.6  -4.9  -4.5  -4.3
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
 -5.8  -6.0  -6.5  -7.4  -8.8 -10.9 -14.1 -19.9 -56.6 -20.2 -14.3 -11.0  -8.9  -7.5  -6.6  -6.0  -5.9
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
 -4.3  -4.4  -4.9  -5.6  -6.7  -8.3 -10.6 -14.0 -19.9 -56.2 -20.5 -14.8 -11.7  -9.8  -8.6  -8.0  -7.8
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
 -3.0  -3.2  -3.5  -4.2  -5.1  -6.4  -8.2 -10.6 -14.1 -20.2 -55.3 -21.3 -15.7 -12.9 -11.2 -10.4 -10.1
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
 -2.0  -2.1  -2.5  -3.0  -3.8  -4.9  -6.4  -8.3 -10.9 -14.6 -20.9 -54.0 -22.5 -17.2 -14.6 -13.4 -13.0
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
 -1.2  -1.3  -1.6  -2.1  -2.9  -3.8  -5.1  -6.7  -8.8 -11.6 -15.5 -22.0 -52.8 -24.3 -19.3 -17.3 -16.7
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
  -.6   -.7  -1.0  -1.5  -2.1  -3.0  -4.2  -5.6  -7.4  -9.7 -12.7 -16.9 -23.6 -51.7 -27.1 -22.7 -21.6
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
  -.2   -.3   -.6  -1.0  -1.6  -2.5  -3.5  -4.9  -6.5  -8.6 -11.1 -14.4 -18.9 -26.1 -50.9 -31.7 -28.9
    X     Y     X     X     X     X     X     X     X     X     X     X     X     X     X     X
   .0   -.0   -.3   -.7  -1.3  -2.1  -3.2  -4.4  -6.0  -7.9 -10.3 -13.2 -16.9 -22.1 -29.9 -50.4 -43.4
    X     X     X     X     X     X     X     X     X     X     X     X     X     X     X     X
  -.0    .0   -.2   -.6  -1.2  -2.0  -3.0  -4.3  -5.8  -7.7 -10.0 -12.8 -16.4 -21.0 -27.6 -37.8 -50.2
```

λ/2

SOURCE

a. Amplitude

Fig. 4. Quadruple size 16 x 16 element amplitude/phase distribution for the room of Fig. 1 at the first room mode (4 x expansion of Fig. 2; with 16 times the number of elements). The exact analytical amplitude distribution for this frequency is: $|p| = P(x, y) = \frac{1}{2}\left(\cos\frac{\pi}{\ell}x + \cos\frac{\pi}{\ell}y\right)$ where ℓ is the length of a side wall. The phase should be 0° on the source side of the node line and 180° on the other. This computer generated distribution is quite close to the theoretical values. a. Amplitude plot. b. Phase plot.

PHASE

DEGS.

```
********** QUICK PRINT PHASE DISTRIBUTION **************
                PHASE IN DEGREES
 -1  179  179  179  179  179  179  179  179  179  179  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1  179  179  179  179  179  179  179  179  179  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1  179  179  179  179  179  179  179  179  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1  179  179  179  179  179  179  179  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1  179  179  179  179  179  179  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1  179  179  179  179  179  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1  179  179  179  179  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1   -1  179  179  179  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1   -1   -1  179  179  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1   -1   -1   -1  179  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1  179  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1  179  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1  179  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1  179  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1  179  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1  179
    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X    X
 -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1   -1
```

b. Phase

Fig. 4 (Continued)

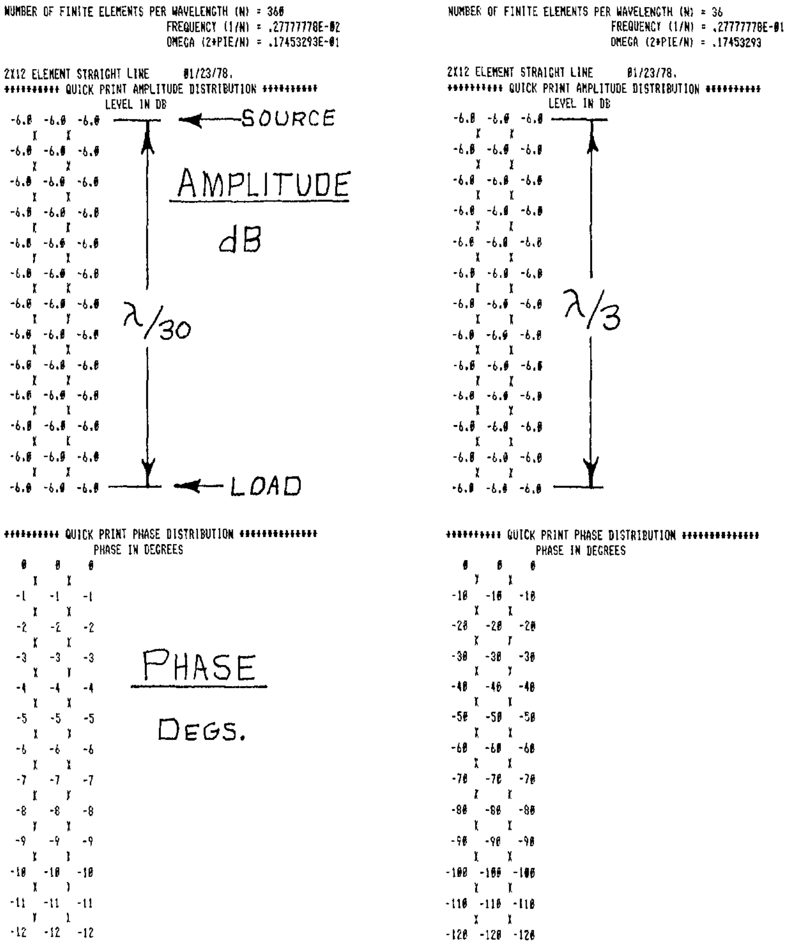
```
REGION DIMENSIONS = 13 NODES DOWN AND 3 NODES ACROSS.

2X12 ELEMENT STRAIGHT LINE      01/23/78.
++++++++++++++ UNCHECKED INPUT IMAGE ++++++++++++++
 $     $     $    <-- SOURCE
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 #     #     #    <-- LOAD

++++++++++++++++++++ SOURCE/LOAD LIST ++++++++++++++++++++
IDENT.  TYPE          SOURCE                  IMPEDANCE
              MAGNITUDE     PHASE     MAGNITUDE     PHASE
                            DEGS.                   DEGS.
  $    SOURCE    1.000000      .0      1.000000        .0
  #     LOAD  -------------  ------    1.000000        .0


NUMBER OF FINITE ELEMENTS = 24
          NUMBER OF NODES = 39
           HALF BANDWIDTH = 4
```

Fig. 5. Computer input image for the 2 x 12 element straight pipe with matched plane wave source "$" at one end and matched load "#" at the other.



a. $f = \frac{1}{360}$ wavelengths per element (wpe)

b. $f = \frac{1}{36}$ wpe

Fig. 6. Computed pressure magnitude and phase distributions for the straight pipe of Fig. 5 at several frequencies. A perfect solution yields an exact pressure of 0.5 or -6.0 dB at all points of the pipe for all frequencies with a linear phase lag as the load is approached.

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 16
                          FREQUENCY (1/N) = .625E-01
                          OMEGA (2*PIE/N) = .39269908

2X12 ELEMENT STRAIGHT LINE      01/23/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                  LEVEL IN DB
 -5.9  -5.9  -5.9
    I     I
 -5.9  -5.9  -5.9
    I     I
 -5.9  -5.9  -5.9
    I     I
 -6.0  -6.0  -6.0
    I     I
 -6.0  -6.0  -6.0
    I     I
 -6.0  -6.0  -6.0
    I     I
 -5.9  -5.9  -5.9
    I     I
 -5.9  -5.9  -5.9        3λ/4
    I     I
 -5.9  -5.9  -5.9
    I     I
 -5.9  -5.9  -5.9
    I     I
 -5.9  -5.9  -5.9
    I     I
 -6.0  -6.0  -6.0
    I     I
 -6.0  -6.0  -6.0

********** QUICK PRINT PHASE DISTRIBUTION **************
                PHASE IN DEGREES
   0     0     0
    I     I
 -22   -22   -22
    I     I
 -45   -45   -45
    I     I
 -68   -68   -68
    I     I
 -91   -91   -91
    I     I
-114  -114  -114
    I     I
-136  -136  -136
    I     I
-159  -159  -159
    I     I
 179   179   179
    I     I
 157   157   157
    I     I
 134   134   134
    I     I
 111   111   111
    I     I
  88    88    88
```

c. $f = \frac{1}{16}$ wpe

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 8
                          FREQUENCY (1/N) = .125
                          OMEGA (2*PIE/N) = .78539816

2X12 ELEMENT STRAIGHT LINE      01/23/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                  LEVEL IN DB
 -6.0  -6.0  -6.0
    I     I
 -5.8  -5.8  -5.8
    I     I
 -5.3  -5.3  -5.3
    I     I
 -5.5  -5.5  -5.5
    I     I
 -6.0  -6.0  -6.0
    I     I
 -5.8  -5.8  -5.8
    I     I
 -5.3  -5.3  -5.3        3λ/2
    I     I
 -5.6  -5.6  -5.6
    I     I
 -6.0  -6.0  -6.0
    I     I
 -5.7  -5.7  -5.7
    I     I
 -5.3  -5.3  -5.3
    I     I
 -5.6  -5.6  -5.6
    I     I
 -6.0  -6.0  -6.0

********** QUICK PRINT PHASE DISTRIBUTION **************
                PHASE IN DEGREES
   1     1     1
    I     I
 -48   -48   -48
    I     I
 -94   -94   -94
    I     I
-137  -137  -137
    I     I
 176   176   176
    I     I
 126   126   126
    I     I
  82    82    82
    I     I
  39    39    39
    I     I
 -10   -10   -10
    I     I
 -59   -59   -59
    I     I
-103  -103  -103
    I     I
-146  -146  -146
    I     I
 165   165   165
```

d. $f = \frac{1}{8}$ wpe

Fig. 6 (Continued)

```
REGION DIMENSIONS = 9 NODES DOWN AND 6 NODES ACROSS.

2X12 ELEMENT RIGHT ANGLE DUCT        01/23/78.
############### UNCHECKED INPUT IMAGE ###############
 $     $     $   <----- SOURCE
    X     X
 0     +     0
    X     X
 0     +     0
    X     X
 0     +     0
    X     X                      LOAD
 0     +     0                     |
    X     X                        |
 0     +     0                     v
    X     X
 0     +     0     0     0     #
    X     X     X     X     X
 0     +     +     +     +     #
    X     X     X     X     X
 0     0     0     0     0     #

##################### SOURCE/LOAD LIST #####################
IDENT.  TYPE              SOURCE                      IMPEDANCE
                  MAGNITUDE     PHASE         MAGNITUDE     PHASE
                                DEGS.                       DEGS.
  $    SOURCE      1.000000        .0          1.000000        .0
  #     LOAD    -------------   ------         1.000000        .0


NUMBER OF FINITE ELEMENTS = 22
          NUMBER OF NODES = 36
           HALF BANDWIDTH = 7
```

Fig. 7. Computer input image for a 2 x 12 element right angle bent pipe with matched source and load.

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 800
                             FREQUENCY (1/N) = .125E-02
                          OMEGA (2*PIE/N) = .78539816E-02

2X12 ELEMENT RIGHT ANGLE DUCT        01/23/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                  LEVEL IN DB
 -6.0  -6.0  -6.0  <-------- SOURCE
    X     X
 -6.0  -6.0  -6.0           AMPLITUDE
    X     X
 -6.0  -6.0  -6.0              dB
    X     X
 -6.0  -6.0  -6.0
    X     X                                λ/100
 -6.0  -6.0  -6.0
    X     X                   LOAD
 -6.0  -6.0  -6.0
    X     X
 -6.0  -6.0  -6.0  -6.0  -6.0  -6.0
    X     X     X     X     X
 -6.0  -6.0  -6.0  -6.0  -6.0  -6.0
    X     X     X     X     X
 -6.0  -6.0  -6.0  -6.0  -6.0  -6.0

********** QUICK PRINT PHASE DISTRIBUTION **************
                PHASE IN DEGREES
   0     0     0
    X     X
  -1    -1    -1
    X     X                 PHASE
  -1    -1    -1
    X     X
  -2    -2    -2             DEGS.
    X     X
  -2    -2    -2
    X     X
  -2    -2    -3
    X     X
  -3    -3    -3    -4    -4    -5
    X     X     X     X     X
  -3    -3    -3    -4    -4    -5
    X     X     X     X     X
  -3    -3    -3    -4    -4    -5
```

a. $f = \frac{1}{800}$ wavelengths per element (wpe)

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 80
                             FREQUENCY (1/N) = .125E-01
                          OMEGA (2*PIE/N) = .78539816E-01

2X12 ELEMENT RIGHT ANGLE DUCT        01/23/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                  LEVEL IN DB
 -6.4  -6.4  -6.4
    X     X
 -6.3  -6.3  -6.3
    X     X
 -6.3  -6.3  -6.3
    X     X
 -6.2  -6.2  -6.2
    X     X                                λ/10
 -6.2  -6.2  -6.2
    X     X
 -6.1  -6.1  -6.1
    X     X
 -6.0  -6.1  -6.1  -6.0  -6.0  -6.0
    X     X     X     X     X
 -6.0  -6.0  -6.0  -6.0  -6.0  -6.0
    X     X     X     X     X
 -6.0  -6.0  -6.0  -6.0  -6.0  -6.0

********** QUICK PRINT PHASE DISTRIBUTION **************
                PHASE IN DEGREES
  -1    -1    -1
    X     X
  -6    -6    -6
    X     X
 -11   -11   -11
    X     X
 -16   -16   -16
    X     X
 -20   -20   -20
    X     X
 -25   -25   -25
    X     X
 -28   -29   -32   -38   -42   -47
    X     X     X     X     X
 -31   -32   -34   -38   -43   -47
    X     X     X     X     X
 -32   -32   -35   -38   -43   -47
```

b. $f = \frac{1}{80}$ wpe

Fig. 8. Pressure and phase distribution for the bent pipe of Fig. 7 at several frequencies. The bend is found to greatly effect the transmission characteristics of the pipe at higher frequencies. Note standing waves in the input pipe for the two highest frequencies. Note also the pressure variation across the pipe at points at or near the bend for the highest frequencies.

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 32
                        FREQUENCY (1/N) = .3125E-01
                        OMEGA (2*PIE/N) = .19634954

2X12 ELEMENT RIGHT ANGLE DUCT       01/23/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                  LEVEL IN DB
+ -6.4  -6.4  -6.4
     X     X
 -6.7  -6.7  -6.7
     X     X
 -6.9  -6.9  -7.0
     X     X
 -7.0  -7.0  -7.0
     X     X
 -6.8  -6.9  -6.9
     X     X
 -6.5  -6.6  -6.6
     X     X
 -6.2  -6.2  -6.3  -6.2  -6.1  -6.1
     X     X     X     X     X
 -5.9  -5.9  -6.0  -6.1  -6.1  -6.1
     X     X     X     X     X
 -5.8  -5.8  -5.9  -6.0  -6.0  -6.1

********** QUICK PRINT PHASE DISTRIBUTION **************
                 PHASE IN DEGREES
   6     6     6
     X     X
  -7    -7    -7
     X     X
 -21   -21   -21
     X     X
 -34   -35   -35
     X     X
 -48   -48   -49
     X     X
 -61   -61   -63
     X     X
 -71   -73   -79   -94  -106  -118
     X     X     X     X     X
 -77   -79   -85   -95  -107  -118
     X     X     X     X     X
 -79   -81   -87   -96  -107  -118
```

λ/4

c. $f = \frac{1}{32}$ wpe

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 16
                        FREQUENCY (1/N) = .625E-01
                        OMEGA (2*PIE/N) = .39269908

2X12 ELEMENT RIGHT ANGLE DUCT       01/23/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                  LEVEL IN DB
 -4.4  -4.4  -4.4
     X     X
 -4.2  -4.2  -4.2
     X     X
 -4.8  -4.8  -4.8
     X     X
 -6.1  -6.2  -6.2
     X     X
 -7.6  -7.7  -7.9
     X     X
 -7.9  -8.1  -8.5
     X     X
 -6.6  -6.8  -7.0  -6.6  -6.4  -6.3
     X     X     X     X     X
 -5.3  -5.5  -5.9  -6.1  -6.2  -6.3
     X     X     X     X     X
 -4.9  -5.1  -5.5  -5.9  -6.1  -6.2

********** QUICK PRINT PHASE DISTRIBUTION **************
                 PHASE IN DEGREES
  -6    -6    -6
     X     X
 -20   -20   -20
     X     X
 -35   -35   -35
     X     X
 -55   -55   -55
     X     X
 -82   -82   -82
     X     X
-115  -117  -120
     X     X
-142  -146  -159   170   145   122
     X     X     X     X     X
-156  -159  -171   168   145   122
     X     X     X     X     X
-159  -163  -174   167   145   122
```

λ/2

d. $f = \frac{1}{16}$ wpe

Fig. 8 (continued)

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 10.6667
                        FREQUENCY (1/N) = .93749707E-01
                        OMEGA (2*PIE/N) = .58904678

2X12 ELEMENT RIGHT ANGLE DUCT       01/23/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                 LEVEL IN DB
 -9.8  -9.8  -9.8
    X     X
 -7.4  -7.4  -7.4
    X     X
 -3.8  -3.8  -3.8
    X     X
 -2.9  -2.9  -2.9
    X     X
 -4.5  -4.6  -4.7
    X     X
 -8.2  -8.7  -9.8
    X     X
 -7.8  -8.4  -9.0  -7.2  -6.8  -6.8
    X     X     X     X     X
 -4.6  -5.0  -5.8  -6.2  -6.5  -6.7
    X     X     X     X     X
 -3.5  -3.9  -4.8  -5.7  -6.4  -6.6
```

3λ/4

```
********** QUICK PRINT PHASE DISTRIBUTION **************
               PHASE IN DEGREES
   9     9     9
    X     X
 -57   -57   -57
    X     X
 -84   -84   -84
    X     X
-100  -100  -100
    X     X
-118  -117  -117
    X     1
-154  -154  -154
    X     X
 146   140   115    67    32    -3
    X     X     X     X     X
 121   115    97    67    33    -3
    X     X     X     X     X
 115   110    94    67    33    -2
```

e. $f = \frac{1}{10.7}$ wpe

```
NUMBER OF FINITE ELEMENTS PER WAVELENGTH (N) = 8
                        FREQUENCY (1/N) = .125
                        OMEGA (2*PIE/N) = .78539816

2X12 ELEMENT RIGHT ANGLE DUCT       01/23/78.
********** QUICK PRINT AMPLITUDE DISTRIBUTION **********
                 LEVEL IN DB
 -2.1  -2.1  -2.1
    X     X
 -4.4  -4.4  -4.4
    X     X
-12.9 -12.8 -12.8
    X     X
 -4.7  -4.7  -4.7
    X     X
 -2.0  -2.0  -2.1
    X     X
 -4.6  -4.9  -5.6
    X     X
 -9.9 -11.9 -15.4  -8.6  -7.6  -7.8
    X     X     X     X     X
 -4.2  -5.0  -6.6  -6.9  -7.4  -7.8
    X     X     X     X     X
 -2.1  -2.8  -4.5  -6.0  -7.2  -7.8
```

λ

```
********** QUICK PRINT PHASE DISTRIBUTION **************
               PHASE IN DEGREES
  -3    -3    -3
    X     X
 -18   -18   -18
    X     X
 -89   -89   -89
    X     X
-168  -168  -167
    X     X
 175   176   177
    X     X
 156   159   163
    X     X
  87    80    26   -49   -89  -137
    X     X     X     X     X
  34    26     0   -42   -86  -134
    X     X     X     X     X
  26    19    -2   -39   -83  -133
```

f. $f = \frac{1}{8}$ wpe

Fig. 8 (continued)

```
REGION DIMENSIONS = 8 NODES DOWN AND 4 NODES ACROSS.

HELMHOLST RESONATOR DUCT EXCITATION      01/27/78.
************** UNCHECKED INPUT IMAGE **************
    A   A  <-- SOURCE
      X
    0   0
      X
    0   0       DUCT
      X
    0   0
      X
0   0   0   0
  X   X   X
0   +   +   0
  X   X   X         CAVITY
0   +   +   0
  X   X   X
0   0   0   0

******************** SOURCE/LOAD LIST ********************
IDENT.  TYPE          SOURCE                   IMPEDANCE
              MAGNITUDE     PHASE       MAGNITUDE     PHASE
                            DEGS.                     DEGS.
  A     SOURCE   1.000000     .0          .100000       .0


NUMBER OF FINITE ELEMENTS = 13
          NUMBER OF NODES = 24
           HALF BANDWIDTH = 5
```

Fig. 9. Input image required for a Helmholtz resonator with source at end of duct. The image describes a 3 x 3 element square cavity coupled to a 1 x 4 element duct or pipe.

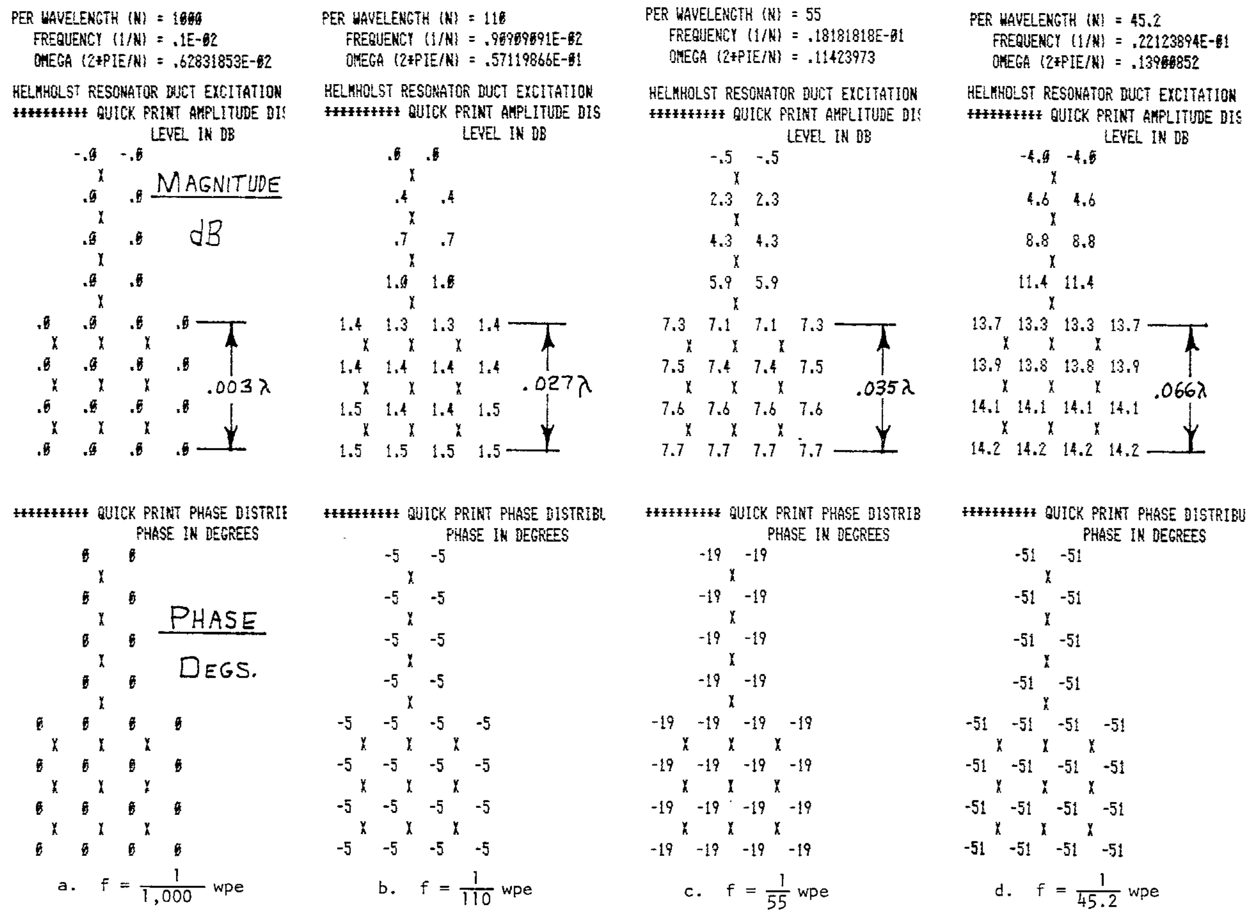


Fig. 10. Magnitude and phase distributions for the Helmholtz resonator of Fig. 9 at several frequencies. Resonance occurs roughly at f = 0.024 or 41.5 elements per wavelength where the cavity pressure is some 16 dB above the source. Note that the program properly predicts the standing waves which occur at higher frequencies.

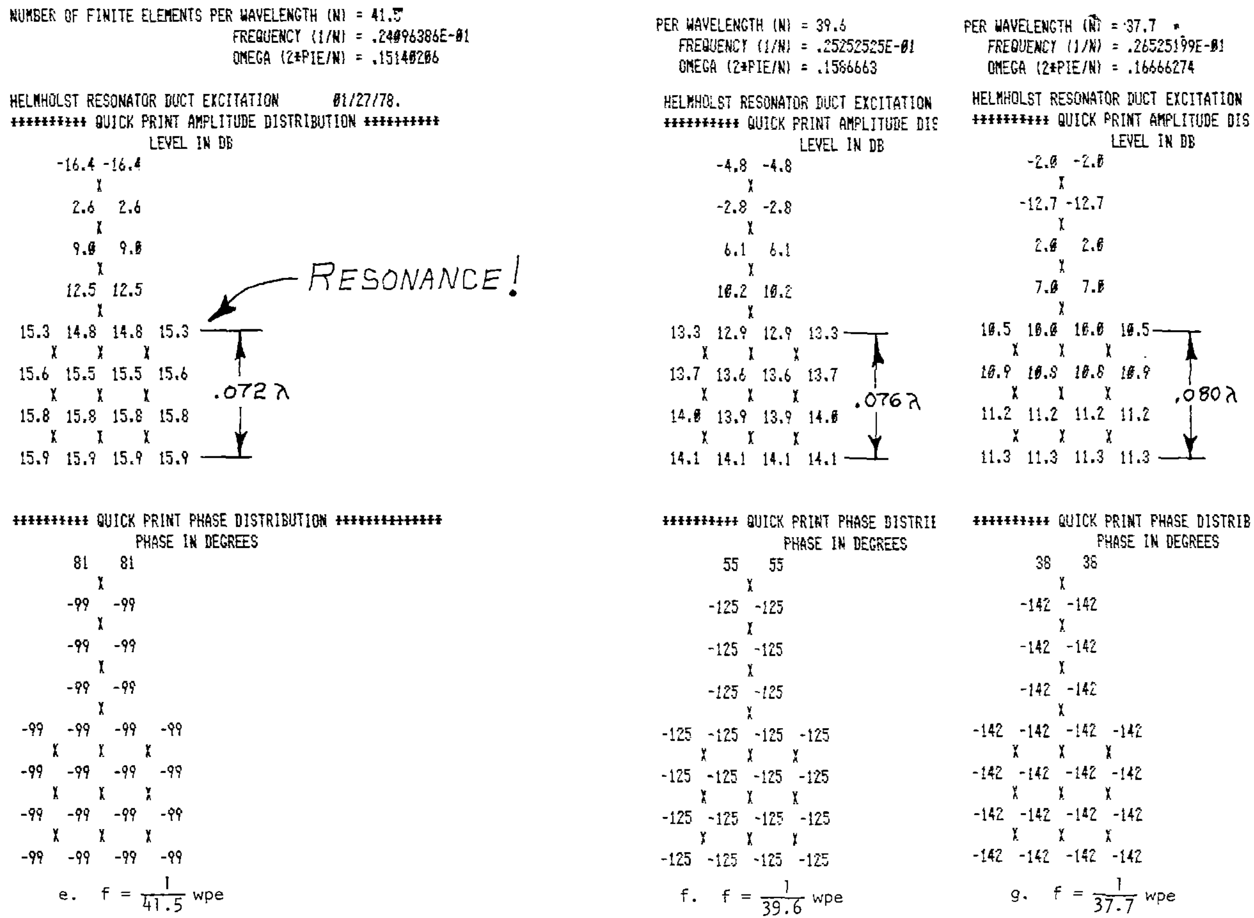


Fig. 10 (continued)

```
PER WAVELENGTH (N) = 30
   FREQUENCY (1/N) = .33333333E-01
   OMEGA (2*PIE/N) = .20943951
HELMHOLST RESONATOR DUCT EXCITATION
********** QUICK PRINT AMPLITUDE DIS
                LEVEL IN DB
        -.2   -.2
            X
        -6.0  -6.0
            X
       -42.5 -42.5
            X
        -6.2  -6.2
            X
   .5   -.3   -.3    .5
      X     X     X
  1.1   1.0   1.0   1.1
      X     X     X          .10 λ
  1.6   1.6   1.6   1.6
      X     X     X
  1.8   1.8   1.8   1.8

********** QUICK PRINT PHASE DISTRIE
               PHASE IN DEGREES
         12    12
            X
         12    12
            X
         12    12
            X
       -168  -168
            X
 -168  -168  -168  -168
      X     X     X
 -168  -168  -168  -168
      X     X     X
 -168  -168  -168  -168
      X     X     X
 -168  -168  -168  -168
```

h. $f = \frac{1}{30}$ wpe

```
PER WAVELENGTH (N) = 25
   FREQUENCY (1/N) = .4E-01
   OMEGA (2*PIE/N) = .25132741
HELMHOLST RESONATOR DUCT EXCITATION
********** QUICK PRINT AMPLITUDE DIS
                LEVEL IN DB
        -.0   -.0
            X
        -3.6  -3.6
            X
       -11.0 -11.0
            X
       -19.0 -19.0
            X
 -4.7  -6.0  -6.0  -4.7
      X     X     X
 -3.8  -4.0  -4.0  -3.8
      X     X     X          .12 λ
 -3.1  -3.1  -3.1  -3.1
      X     X     X
 -2.8  -2.8  -2.8  -2.8

********** QUICK PRINT PHASE DISTRIB
               PHASE IN DEGREES
          7     7
            X
          7     7
            X
          7     7
            X
       -173  -173
            X
 -173  -173  -173  -173
      X     X     X
 -173  -173  -173  -173
      X     X     X
 -173  -173  -173  -173
      X     X     X
 -173  -173  -173  -173
```

i. $f = \frac{1}{25}$ wpe

```
PER WAVELENGTH (N) = 14
   FREQUENCY (1/N) = .71428571E-01
   OMEGA (2*PIE/N) = .44879895
HELMHOLST RESONATOR DUCT EXCITATION
********** QUICK PRINT AMPLITUDE DIS
                LEVEL IN DB
        -.0   -.0
            X
        -.1   -.1
            X
        -2.2  -2.2
            X
        -7.7  -7.7
            X
-17.0 -28.1 -28.1 -17.0
      X     X     X
-12.6 -13.7 -13.7 -12.8
      X     X     X          .21 λ
-10.0 -10.1 -10.1 -10.0
      X     X     X
 -9.1  -9.2  -9.2  -9.1

********** QUICK PRINT PHASE DISTRIBU
               PHASE IN DEGREES
         -1    -1
            X
         -1    -1
            X
         -1    -1
            X
         -1    -1
            X
  179   179   179   179
      X     X     X
  179   179   179   179
      X     X     X
  179   179   179   179
      X     X     X
  179   179   179   179
```

j. $f = \frac{1}{14}$ wpe

```
PER WAVELENGTH (N) = 10
   FREQUENCY (1/N) = .1
   OMEGA (2*PIE/N) = .62831853
HELMHOLST RESONATOR DUCT EXCITATION
********** QUICK PRINT AMPLITUDE DIS
                LEVEL IN DB
        -.2   -.2
            X
         6.4   6.4
            X
         7.6   7.6
            X
         4.8   4.8
            X
-24.7  -8.0  -8.0 -24.7
      X     X     X
-10.6 -13.3 -13.3 -10.6
      X     X     X          .30 λ
 -3.2  -3.5  -3.5  -3.2
      X     X     X
 -1.4  -1.5  -1.5  -1.4

********** QUICK PRINT PHASE DISTRI
               PHASE IN DEGREES
        -12   -12
            X
        -12   -12
            X
        -12   -12
            X
        -12   -12
            X
  -12   -12   -12   -12
      X     X     X
  168   168   168   168
      X     X     X
  168   168   168   168
      X     X     X
  168   168   168   168
```

k. $f = \frac{1}{10}$ wpe

Fig. 10 (continued)