DIRECT LOW-FREQUENCY DRIVER SYNTHESIS
FROM SYSTEM SPECIFICATIONS

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DIRECT LOW-FREQUENCY DRIVER SYNTHESIS FROM SYSTEM SPECIFICATIONS

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The usual procedure for direct-radiator low-frequency loudspeaker system design leads to calculation of the driver's fundamental electromechanical parameters by an intermediate specification of the Thiele/Small parameters. A reformulation of the synthesis procedure to eliminate the intermediate Thiele/Small calculation leads to a set of equations that yield the driver's electromechanical parameters directly from the system specifications.

These equations reveal some moderately surprising relationships when the different system types (closed-box, fourth-order vented-box, sixth-order vented-box) are compared. For example, for a specified LF cutoff (f_3), midband efficiency and driver size, the fourth-order vented-box driver is found to be roughly three times more expensive (judged on the amount of magnet energy required) than the closed-box driver. Conversely for a given f_3, enclosure volume (V_B), maximum diaphragm excursion (x_{max}) and acoustic power output (P_{AR}) the fourth-order vented-box driver is some five times cheaper than the closed-box driver!

It is also found that for direct-radiator systems in general, a specified f_3, V_B, x_{max}, and P_{AR} leads to the total moving mass (M_{MS}) depending inversely on the sixth power of the cutoff frequency i.e. a one-third-octave reduction in f_3 results in a four fold increase in mass! Furthermore, the same conditions reveal that the sixth-order vented-box driver moving mass is some 42 times lighter than the closed-box driver providing the same midband acoustic output and f_3! If cone area and efficiency are held constant, the direct-radiator system driver actually gets less expensive as the low-frequency limit is extended.

GLOSSARY OF SYMBOLS

B	magnetic flux density in driver air gap
c	velocity of sound in air (= 345 m/s)
C_{MS}	compliance of driver diaphragm suspension
f	frequency (Hz)
f_B	resonance frequency of vented enclosure
f_C	resonance frequency of closed-box system

f_S	resonance frequency of unenclosed driver
f_3	low-end cutoff (half-power or -3 dB) frequency of system
h	system tuning ratio (= f_B/f_S)
k_P	power rating constant
k_3	frequency ratio constant for system (= f_S/f_3)
k_η	efficiency constant (assumes η_o in %)
l	length of voice-coil conductor in magnetic gap
M_{MS}	total moving mass of driver including air loads
P_{AR}	displacement-limited acoustic power output rating (used in this paper in a more general sense to indicate the midband maximum acoustic output of a system; drivers designed from the equations in this paper will reach their displacement and thermally limited input limits at the same continuous power levels in the system passband ($P_{ER} = P_{E(max)}$)
P_{ER}	displacement-limited electrical power rating
$P_{E(max)}$	thermally-limited maximum input power
Q_B	total enclosure Q at f_B resulting from all enclosure and vent losses
Q_L	enclosure Q at f_B resulting from leakage losses
Q_T	total Q of driver at f_S resulting from all system resistances
Q_{ES}	Q of driver at f_S considering electromagnetic damping only
Q_{MS}	Q of driver at f_S considering mechanical losses only
Q_{TC}	total Q of closed-box system at f_C including all system resistances
Q_{TS}	total Q of driver at f_S considering all driver resistances $\left(= \dfrac{Q_{ES}\,Q_{MS}}{Q_{ES}+Q_{MS}}\right)$
R_E	dc resistance of driver voice coil
R_{ME}	electromagnetic damping factor of driver (= $(Bl)^2/R_E$)
S_D	effective surface area of driver diaphragm
V_{AS}	volume of air having same acoustic compliance as driver suspension (compliance equivalent volume)
V_B	net internal volume of enclosure
V_D	peak displacement volume of driver diaphragm (= $S_D\,x_{max}$)
x_{max}	peak linear displacement of driver diaphragm
α	system compliance ratio (= V_{AS}/V_B)
η_o	reference efficiency of system in % (half-space acoustic load)
ρ_o	density of air (= 1.21 kg/m^3)
ω	radian frequency (= $2\pi f$)

1. INTRODUCTION: Traditionally (pre 1970) the design of direct-radiator loudspeaker systems has been mostly an empirical process. Quoting R.H. Small in his introduction to his monumental series of papers on direct-radiator loudspeaker system design [1-3]:

> "The design of a loudspeaker system is traditionally a trial-and-error process guided by experience: a likely driver is chosen and various enclosure designs are tried until the system performance is found to be satisfactory. In sharp contrast to this empirical design process is the synthesis of many other engineering systems. This begins with the desired system performance specifications and leads directly to specification of system components. This latter approach requires the engineer to have precise knowledge of the relationships between system performance and component specifications."

Small's synthesis techniques for closed-box [2] and vented-box [3] system design start from the desired system specifications such as midband reference efficiency (η_o), net internal enclosure volume (V_B), low frequency cutoff point (f_3), midband maximum acoustic output power (P_{AR}) and leads to specification of the required driver in terms of the basic design parameters called the Thiele/Small parameters f_S, V_{AS}, Q_{ES}, Q_{MS}, η_o, V_D and $P_{E(max)}$. These Thiele/Small driver parameters are then used with a selected driver diaphragm area (S_D) or maximum diaphragm excursion value (x_{max}), and desired voice-coil resistance (R_E) to compute the drivers fundamental electromechanical parameters: total moving mass (M_{MS}), suspension compliance (C_{MS}), Bl product, and electromagnetic damping factor (R_{ME}), which being roughly proportional to magnet assembly weight is also proportional to magnet cost.

Even though the concept of the Thiele/Small driver parameters has contributed greatly to the analysis, synthesis, design, and measurement of direct-radiator loudspeaker systems, the intermediate calculation of these parameters somewhat disguises the important relationships between the driver's mechanical parameters and the target system specifications. In this paper, equations are developed which yield the driver's fundamental electromechanical parameters directly from the desired system specifications. Important relationships are derived which show how the driver's electromagnetic damping factor (and hence magnet cost) depends on system type (closed-box vs vented-box vs equalized vented-box) and specifications.

2. DRIVER DESIGN VIA INTERMEDIATE THIELE/SMALL PARAMETERS

Small's design techniques for direct-radiator loudspeaker systems start from a specification of the desired performance required which includes:

f_3	low-frequency half-power (-3 dB) cutoff point,
V_B	net internal volume of enclosure,
η_o	midband reference efficiency (half-space load),
Response shape:	i.e. system type and alignment information, and
P_{AR}	maximum midband acoustic output power

(note that only two of the three specifications f_3, V_B, and η_o can be selected, the third depends on the first two through the efficiency equation: $\eta_o = k_\eta f_3^3 V_B$).

Once the system type (closed, vented or vented plus EQ, etc.) and frequency response shape have been selected, values for the system parameters k_3 (= f_S/f_3), α (= V_{AS}/V_B), Q_T, h (= f_B/f_S), and k_P are fixed and can be determined by a number of different means (Small [2,3], Thiele [4], and Keele [5]. Knowledge of the systems large-signal target specifications along with the system parameters allows calculation of the drivers Thiele/Small basic design parameters f_S, Q_{ES}, Q_{MS}, Q_{TS}, V_{AS}, V_D, and $P_{E(max)}$ as follows:

$$f_S = k_3 f_3 \quad (1)$$
$$V_{AS} = \alpha V_B \quad (2)$$
$$Q_{TS} = Q_T \quad (3)$$
$$Q_{MS} = \text{Selected,}$$
$$Q_{ES} = Q_{MS} Q_{TS} / (Q_{MS} - Q_{TS}), \text{ and} \quad (4)$$
$$\eta_o = k_\eta f_3^3 V_B. \quad (5)$$

Small's relationship for the displacement limited acoustic output power [2, eq. 40]:

$$P_{AR} = k_P f_3^4 V_D^2 \quad (6)$$

can be solved for V_D yielding:

$$V_D = \frac{1}{\sqrt{k_P}} \cdot \frac{\sqrt{P_{AR}}}{f_3^2}, \text{ and} \quad (7)$$

$$P_{ER} = P_{AR}/\eta_o \quad (8)$$

The physical specification of the driver may now be completed by selecting arbitrarily the driver cone area S_D and voice coil resistance R_E, and then calculating the driver's electromechanical parameters as outlined in [2, sec. 10]. The required value of peak displacement volume (V_D) for the driver must be divided into acceptable values of S_D and x_{max} (S_D may be arbitrarily selected or may be computed knowing the desired value of x_{max} i.e. $S_D = V_D/x_{max}$).

The driver's electromechanical parameters are given by [2, eqs. 61 - 65]:

$$C_{MS} = \frac{V_{AS}}{\rho_o c^2 S_D^2} \quad (9)$$

$$M_{MS} = \frac{1}{(2\pi f_S)^2 C_{MS}} \quad (10)$$

$$Bl = \sqrt{\frac{2\pi f_S R_E M_{MS}}{Q_{ES}}} \tag{11}$$

$$R_{ME} = (Bl)^2/R_E = \frac{2\pi f_S M_{MS}}{Q_{ES}} \tag{12}$$

3. DIRECT DRIVER SYNTHESIS

Method

The design method outlined in the previous section can be streamlined by eliminating the intermediate calculation of Thiele/Small parameters and proceeding directly from the system specifications to the driver's mechanical parameters. In general, substitutions are made in eqs. (9) - (12) for the Thiele/Small parameters in terms of the system specifications and a set of equations are derived yielding the desired driver mechanical parameters.

System Specifications Required

The derived equations can be organized in several different ways depending on which of the independent system specifications are chosen. I have chosen to derive three complete sets of equations each using as variables two of the three parameters of the efficiency-volume-cutoff set (η_o, V_B, f_3). Within each of the three sets a further division is made on roughly small-signal/large-signal considerations. In the first small-signal category, S_D is chosen arbitrarily and appears explicitly in the equations while x_{max} is allowed to float so that V_D is satisfied. In the second category, the values for P_{AR}, x_{max}, and k_P appear explicitly with the cone area S_D allowed to float.

Derivation Example

To illustrate the derivations of the equations, one example will be worked out for one pair of specifications (f_3, V_B) to yield the equations for the total moving mass M_{MS}.

From a knowledge of the system parameters k_3, α, Q_T, H, k_η, and k_P derived from knowing the desired system type and response function (see Sec. 2) the following derivation can be worked out. From the knowledge that

$$V_{AS} = \alpha V_B \tag{13}$$

eq. (9) can be transformed into:

$$C_{MS} = \frac{\alpha}{\rho_o c^2} \cdot \frac{V_B}{S_D^2} \tag{14}$$

This value of C_{MS} along with

$$f_S = k_3 f_3 \tag{15}$$

when substituted into eq. (10) yields:

$$M_{MS} = \frac{\rho_o\ c^2}{4\ \pi^2\ k_3^2 \alpha} \cdot \frac{S_D^2}{f_3^2\ V_B} \qquad (16)$$

which is the desired result with mostly small-signal specifications evident and S_D appearing explicitly.

To convert to the large-signal format, Small's relationship for the displacement limited acoustic output power eq. (6) must be used along with

$$V_D = S_D\ x_{max} \qquad (17)$$

to yield a value for S_D in terms of mostly large-signal specifications:

$$S_D = \frac{P_{AR}}{x_{max}^2\ k_p f_3^4} \qquad (18)$$

This value for S_D can inturn be used with eq. (16) to give M_{MS} in terms of mostly large-signal system specifications:

$$M_{MS} = \frac{\rho_o\ c^2}{4 \pi^2 k_3^2 \alpha\ k_P} \cdot \frac{P_{AR}}{x_{max}^2\ f_3^6\ V_B} \qquad (19)$$

This equation illustrates the very strong dependence of M_{MS} on f_3 (inversely on the sixth power of f_3!) for a specified P_{AR}, x_{max}, and V_B. Observations on the whole set of derived equations will be deferred until after all the equations are shown.

4. DIRECT SYNTHESIS EQUATIONS

Following the procedure as outlined in Sec. 3, the equations can be derived for all the driver's mechanical parameters. These equations are shown in the following listing.

From Small-Signal Specifications

Combinations

Total Moving Mass

f_3 & η_o $$M_{MS} = \frac{\rho_o\ c^2 k_\eta}{4\pi^2\ k_3^2 \alpha} \cdot \frac{S_D^2\ f_3}{\eta_o} \qquad (20)$$

f_3 & V_B $$= \frac{\rho_o\ c^2}{4\pi^2\ k_3^2 \alpha} \cdot \frac{S_D^2}{f_3^2\ V_B} \qquad (21)$$

η_o & V_B $$= \frac{\rho_o\ c^2\ k_\eta^{2/3}}{4\pi^2\ k_3^2 \alpha} \cdot \frac{S_D^2}{\eta_o^{2/3}\ V_B^{1/3}} \qquad (22)$$

From Small-Signal Specifications (continued)

Combinations

Suspension Compliance

f_3 & η_o
$$C_{MS} = \frac{\alpha}{\rho_o c^2 k_\eta} \cdot \frac{\eta_o}{S_D^2 f_3^3} \qquad (23)$$

f_3 & V_B
$$= \frac{\alpha}{\rho_o c^2} \cdot \frac{V_B}{S_D^2} \qquad (24)$$

η_o & V_B
$$= \frac{\alpha}{\rho_o c^2} \cdot \frac{V_B}{S_D^2} \qquad (25)$$

Bl Product

f_3 & η_o
$$Bl = \sqrt{\frac{\rho_o c^2 k_\eta}{2\pi Q_{ES} k_3 \alpha}} \cdot S_D f_3 \sqrt{\frac{R_E}{\eta_o}} \qquad (26)$$

f_3 & V_B
$$= \sqrt{\frac{\rho_o c^2}{2\pi Q_{ES} k_3 \alpha}} \cdot S_D \sqrt{\frac{R_E}{f_3 V_B}} \qquad (27)$$

η_o & V_B
$$= \sqrt{\frac{\rho_o c^2 k_\eta^{1/3}}{2\pi Q_{ES} k_3 \alpha}} \cdot S_D \sqrt{\frac{R_E}{\eta_o^{1/3} V_B^{2/3}}} \qquad (28)$$

Electro-magnetic Damping

f_3 & η_o
$$R_{ME} = \frac{\rho_o c^2 k_\eta}{2\pi Q_{ES} k_3 \alpha} \cdot \frac{S_D^2 f_3^2}{\eta_o} \qquad (29)$$

f_3 & V_B
$$= \frac{\rho_o c^2}{2\pi Q_{ES} k_3 \alpha} \cdot \frac{S_D^2}{f_3 V_B} \qquad (30)$$

η_o & V_B
$$= \frac{\rho_o c^2 k_\eta^{1/3}}{2\pi Q_{ES} k_3 \alpha} \cdot \frac{S_D^2}{\eta_o^{1/3} V_B^{2/3}} \qquad (31)$$

From Large-Signal Specifications

Combinations

Total Moving Mass

f_3 & η_o $$M_{MS} = \frac{\rho_o c^2 k_\eta}{4\pi^2 k_3^2 \alpha k_p} \cdot \frac{P_{AR}}{x_{max}^2 f_3^3 \eta_o} \quad (32)$$

f_3 & V_B $$= \frac{\rho_o c^2}{4\pi^2 k_3^2 \alpha k_p} \cdot \frac{P_{AR}}{x_{max}^2 f_3^6 V_B} \quad (33)$$

η_o & V_B $$= \frac{\rho_o c^2 k_\eta^2}{4\pi^2 k_3^2 \alpha k_p} \cdot \frac{V_B P_{AR}}{x_{max}^2 \eta_o^2} \quad (34)$$

Suspension Compliance

f_3 & η_o $$C_{MS} = \frac{\alpha k_p}{\rho_o c^2 k_\eta} \cdot \frac{\eta_o f_3 x_{max}^2}{P_{AR}} \quad (35)$$

f_3 & V_B $$= \frac{\alpha k_p}{\rho_o c^2} \cdot \frac{x_{max}^2 f_3^4 V_B}{P_{AR}} \quad (36)$$

η_o & V_B $$= \frac{\alpha k_p}{\rho_o c^2 k_\eta^{4/3}} \cdot \frac{x_{max}^2 \eta_o^{4/3}}{P_{AR} V_B^{1/3}} \quad (37)$$

Bl Product

f_3 & η_o $$Bl = \sqrt{\frac{\rho_o c^2 k_\eta}{2\pi Q_{ES} k_3 \alpha k_p}} \cdot \frac{1}{f_3 x_{max}} \sqrt{\frac{P_{AR} R_E}{\eta_o}} \quad (38)$$

From Large-Signal Specifications (continued)

Combinations

BI Product (continued)

f_3 & V_B

$$= \sqrt{\frac{\rho_o\ c^2}{2\pi Q_{ES} k_3 \alpha k_P}} \cdot \frac{1}{x_{max}} \sqrt{\frac{P_{AR} R_E}{f_3^5 V_B}} \qquad (39)$$

η_o & V_B

$$= \sqrt{\frac{\rho_o\ c^2 k_\eta^{5/3}}{2\pi Q_{ES} k_3 \alpha k_P}} \cdot \frac{1}{x_{max}} \sqrt{\frac{P_{AR} R_E V_B^{2/3}}{\eta_o^{5/3}}} \qquad (40)$$

Electromagnetic Damping

f_3 & η_o

$$R_{ME} = \frac{\rho_o\ c^2 k_\eta}{2\pi Q_{ES} k_3 \alpha k_P} \cdot \frac{P_{AR}}{f_3^2 x_{max}^2 \eta_o} \qquad (41)$$

f_3 & V_B

$$= \frac{\rho_o\ c^2}{2\pi Q_{ES} k_3 \alpha k_P} \cdot \frac{P_{AR}}{f_3^5 x_{max}^2 V_B} \qquad (42)$$

η_o & V_B

$$= \frac{\rho_o\ c^2 k_\eta^{5/3}}{2\pi Q_{ES} k_3 \alpha k_P} \cdot \frac{P_{AR} V_B^{2/3}}{\eta_o^{5/3} x_{max}^2} \qquad (43)$$

5. SELECTION OF SYSTEM TYPE AND RESPONSE SHAPES

Three different types of direct-radiator loudspeaker systems were chosen to illustrate the use of the derived equations:

1. C2ND: Closed-box, second-order high-pass response,
2. V4TH: Vented-box, fourth-order high-pass response, and
3. V6TH: Vented-box, sixth-order high-pass response.

The third system type (V6TH) requires the use of an external second-order high-pass filter/equalizer which provides sub-sonic energy rejection and modest boost equalization (+6 dB) in the lowest octave of operation (see [4, Alignment 15], [5], [6], and [8]. All three system types have been analyzed extensively and enjoy popularity in the market-place [1-8]. Maximally-flat (Butterworth) or near maximally-flat response curves were chosen for each of the three systems (0.4 dB ripple or less).

The system parameters of the three types were selected from information contained in [2-3, 5]. Reasonable system losses were assumed with realistic attainable efficiency constants (k_η). The system parameters were juggled so that an exact ratio of efficiency constants between the system types was maintained, i.e. a ratio of 1:2:5 (0, +3 dB, +7 dB) for the closed-box, vented-box fourth-order, and vented-box sixth-order system types respectively. These efficiency ratios are very close to the ratios attained by the theoretical loss-free systems.

All parameters were derived assuming the systems are operated from amplifiers having negligible output resistance. The closed-box type is further assumed to obtain most of it's total damping from electromagnetic coupling and mechanical driver losses, and the effects of any filling materials were neglected [2, Sec. 9]. The vented-box system is assumed to have a leakage loss Q of ($Q_B = Q_L = 7$) [3, Sec. 11]. The selected system parameters are shown in Table 1. Table 2 shows the Thiele/Small driver design data and equations using the system parameters from Table 1 and the relationships of eqs. (1) - (8). Fig. 1 shows the small-signal response curves of the three chosen system types normalized to their midband efficiency levels.

6. DIRECT SYNTHESIS EQUATIONS WITH EVALUATED CONSTANTS

The system information in Tables 1 and 2 was used to evaluate the constant factors in eqs. (20) - (43) for the three system types. The resultant equations are shown in Tables 3 - 5.

7. EQUATION OBSERVATIONS

A number of moderately surprising observations can be made about the direct synthesis equations shown in Tables 3 - 5. Some rather interesting relationships are indicated when the different system types are compared. A number of these relationships have been observed before but it helps to repeat them here as an aid to the system designer for gaining insight into system design factors and tradeoffs [2 - 8]. Refer to Tables 3 - 5 when reading the following outlined observations.

General Observations

The following comments apply to all the analyzed system types.

A. Small-Signal

1. The total driver moving mass (M_{MS}) and R_{ME} (cost) are proportional to the square of the cone area (S_D) or cone diameter to the fourth power (a strong dependence!). Large cones mean lots of mass to move around and lots of magnet to move them! If possible, it would be more economical to reduce M_{MS} and R_{ME} by trading S_D for x_{max} so that V_D is maintained ($V_D = S_D\, x_{max}$).

2. Bl is directly proportional S_D.

3. Suspension compliance (C_{MS}) goes as $1/S_D^2$ or $1/\text{dia}^4$. Large cones imply not only high mass but also require high stiffness for control.

B. Large-signal

1. M_{MS} and R_{ME} (cost) are proportional to P_{AR} and inversely proportional to x_{max}^2. This shows the magnet cost rising in direct proportion to the desired acoustic output power and decreasing with the square of the drivers excursion capabilities. You will pay for high power output! Any allowable increase in $x_{max.}$ will pay for itself in decreased driver cost.

2. Compliance goes as $1/P_{AR}$. High driver suspension stiffness goes along with high power drivers. Most high-power musical instrument and professional loudspeakers have suspensions which are quite stiff.

3. Bl goes as $1/x_{max}$ and the square root of P_{AR}.

Specific Observations

These observations apply only to the specific system types and particular parameters specified.

A. For Specified Cutoff Frequency and Efficiency (Table 3)

1. If you are forced into using a particular driver size, the closed-box system is cheaper! This is because M_{MS} is higher for the vented systems at the same time the box size is smaller. The following table shows the approximate ratios if S_D is held constant.

Type	M_{MS} & Bl	R_{ME} (cost)
C2ND	x 1	x 1
V4TH	x 1.8	x 3.1
V6TH	x 2.2	x 4.9

2. The vented-box system is cost effective only if the driver size can be allowed to decrease. The table shows approximate cost break-even points for driver size specifications.

Type	Area Ratio	Diameter Ratio
C2ND	1	1
V4TH	0.56	0.75
V6TH	0.45	0.67

This means that a sixth-order vented system driver must be two-thirds the diameter of the closed-box system or smaller to be cost effective.

3. Small-Signal (specified f_3, η_0, and S_D)
   a. M_{MS} and Bl are proportional to f_3. This is true for a fixed S_D and η_0 because as f_3 goes up the box size grows smaller.

b. R_{ME} (cost) varies as f_3^2. In this case the driver actually gets cheaper as the system's response is extended lower (normally it's the other way around if the box size is maintained constant!, see Sec. B.3.d further on for behavior under specified f_3 and V_B). This is due to the box size getting larger as f_3 is decreased.

c. M_{MS} and R_{ME} (cost) varies as $1/\eta_0$. High efficiency costs less!

4. Large-Signal (specified f_3, η_0, P_{AR}, and x_{max} (S_D float))

a. For a specified x_{max} and P_{AR} the vented-box drivers are cheaper because the cone size is smaller.

Type	R_{ME} (cost) Ratio
C2ND	1
V4TH	0.39 (1/2.6)
V6TH	0.26 (1/3.8

b. Vented-box drivers are lighter.

Type	M_{MS} Ratio
C2ND	1
V4TH	0.22 (1/4.6)
V6TH	0.12 (1/8.5)

B. For Specified Cutoff Frequency and Box Volume (Table 4)

1. M_{MS} and R_{ME} (cost) proportional to $1/V_B$. Drivers for small boxes are more expensive!

2. Small-Signal (specified f_3, V_B, and S_D)

a. For the same S_D, the closed-box and sixth-order vented-box drivers are nearly the same cost with the fourth-order vented-box driver more expensive.

Type	R_{ME} (cost) Ratio
C2ND	1
V4TH	1.57
V6TH	0.98

b. Vented-box cones are lighter.

Type	M_{MS} Ratio
C2ND	1
V4TH	0.88 (1/1.13)
V6TH	0.44 (1/2.3)

3. Large-signal (specified f_3, V_B, P_{AR}, and x_{max})
   a. For the same excursion capabilities (x_{max}) the vented-box cones are very much lighter! This is due to the much larger diaphragm sizes for the lower-order system types.

Type	M_{MS} Ratio	
C2ND	1	
V4TH	0.11	(9 times lighter!)
V6TH	0.024	(42 times lighter!!)

   b. Moving mass varies inversely as the sixth power of the cutoff frequency! Driver moving mass increases quite dramatically for decreases in f_3 because the cone size increases also. For each 1/3rd octave extension of bass response the mass goes up by a factor of four!

   c. Vented-box drivers are much cheaper (because the cone sizes are much smaller for a specified x_{max})!

Type	R_{ME} (cost) Ratio	
C2ND	1	
V4TH	0.19	(5 times cheaper!)
V6TH	0.052	(19 times cheaper!)

   d. R_{ME} (cost) varies as $1/f_3^5$. You pay dearly to extend the low-frequency response of the system. For each 1/3rd octave decrease in f_3, cost rises by 3.2! Contrast this with the activity in Sec. A.3.b mentioned previously for specified f_3 and η_o.

C. For Specified Efficiency and Box Volume (Table 5)
1. M_{MS} and R_{ME} (cost) are proportional to $1/\eta_o^x$ where x ranges from 1/3 to 2 for different parameters and conditions. Higher efficiency is cheaper! This is due to cutoff rising with increases in efficiency.

2. Small-Signal (specified η_o, V_B, and S_D)
   a. Vented-box drivers are more expensive if driver size is fixed (see Secs. A.4.a and B.3.c).

Type	R_{ME} (cost) Ratio
C2ND	1
V4TH	2.0
V6TH	1.7

b. Vented-box drivers are heavier if cone size is fixed.

Type	M_{MS} Ratio
C2ND	1
V4TH	1.4
V6TH	1.3

c. R_{ME} (cost) is proportional to $1/V_B^{(2/3)}$. Big box drivers are cheaper if S_D and η_o are fixed.

3. Large-Signal (specified η_o, V_B, P_{AR}, and x_{max})

a. Vented-box drivers are less expensive if S_D is allowed to float (x_{max} constant, contrast this with Sec. C.2.a above).

Type	R_{ME} (cost) Ratio
C2ND	1
V4TH	0.61
V6TH	0.76

b. Vented-box drivers are lighter if S_D is allowed to float (x_{max} constant, contrast this with Sec. C.2.b above).

Type	M_{MS} Ratio
C2ND	1
V4TH	0.44
V6TH	0.59

c. R_{ME} goes as $V_B^{(2/3)}$. Big box drivers are more expensive if x_{max}, η_o, and P_{AR} are fixed (contrast with Sec. C.2.c above).

8. EXAMPLES OF SYSTEM DESIGN

Three sets of examples will be worked out for the three combinations of independent system specifications: 1. f_3 & η_o, 2. f_3 & V_B, and 3. η_o & V_B. For each set of combinations, the three types of systems will be synthesized for comparison purposes. Only brief information will be contained here, please refer to [1], [2, Secs. 9 - 10], [3, Secs. 11 - 12] and [5] for more complete design information. For each example, graphs will be shown plotting the small-signal frequency response (efficiency vs frequency), maximum continuous acoustic output power and maximum continuous electrical input power.

Maximum Acoustic Output Power

The curves for output power indicate, at each frequency, the maximum continuous acoustic power output and sound pressure levels (SPL re 20 μP) generated in the reverberant field of a reference environment (85 m^3 (3000 ft^3) room with 200 sabins of absorption) before 1. the driver burns itself up (driver thermal limit) or 2. distortion becomes too high (driver displacement limit), whichever occurs first [5, Appendix 1]. The low frequency maximum power output of a driver depends highly on the type of enclosure it is used in. Frequency response equalization (if used), of course, has no effect on maximum acoustic output.

Ideally, a system should be thermally limited over its full operating frequency range. Displacement limiting implies that the system's input electrical power must be decreased below the driver's thermally limited maximum input power $P_{E(max)}$, or distortion will become too high. The computer simulations used in this paper assumes that the driver's cone displacement is linear up to $\pm x_{max}$ (distortion acceptable) and non-linear beyond (distortion unacceptable). The system's displacement limited maximum output is the power the system can generate when the cone excursion is $\pm x_{max}$.

Maximum Electrical Input Power

The curves for input power indicate, at each frequency, the maximum continuous electrical input power before 1. the driver burns itself up (driver thermal limit P in = $P_{E(max)}$) or 2. driver displacement equals $\pm x_{max}$ (driver displacement limit), whichever occurs first. Displacement limited input power is indicated by input powers less than the thermal limit (Pin $< P_{E(max)}$). The transition to displacement limiting is usually indicated on the curves by a sharp break with sudden dropoff in maximum input power as the frequency is lowered. Note that the maximum acoustic output power of the system at a particular frequency is simply the maximum electrical input power at that frequency times the efficiency at that frequency.

Example 1. Specify f_3, η_o, S_D and $P_{E(max)}$

A sub-woofer system is to be designed using a 460 mm (18 in) diameter driver with a 100 watt amplifier to go down to 20 Hz. Enclsoure size is unimportant but the system has to be able to generate 0.7 acoustic watt midband (generates roughly 112 dB SPL in the reverberent field of a typical living room, 85 m^3 (3000 ft^3) with R= 200 sabins of absorption).

The following specifications must be met:

f_3	20 Hz
Response:	Near maximally flat acceptable
η_o	0.7% (= P_{AR}/P_{ER})
S_D	0.1134m^2 (176 in^2)
$P_{E(max)}$, P_{ER}	100 W
P_{AR}	0.7 W

The small-signal equations in Table 3 may be used to compute M_{MS}, C_{MS}, Bl, and R_{ME} as follows

(closed-box system type (C2ND), $R_E = 6.5$ ohms):

$$M_{MS} = 0.45 \cdot \frac{S_D^2 f_3}{\eta_o} = \frac{0.45\,(0.1134)^2 20}{0.7} = 0.17 \text{ kg},$$

$$C_{MS} = 0.297 \cdot \frac{\eta_o}{S_D^2 f_3} = \frac{0.297\,(0.7)}{(0.1134)^2\,(20)^3} = 2.0 \times 10^{-3} \text{ m/N},$$

$$Bl = 1.93 \cdot S_D f_3 \sqrt{\frac{R_E}{\eta_o}} = 1.93\,(0.1134)\,(20)\sqrt{\frac{6.5}{0.7}} = 13.3 \text{ T}\cdot\text{m, and}$$

$$R_{ME} = 3.71 \cdot \frac{S_D^2 f_3^2}{\eta_o} = \frac{3.71\,(0.1134)^2\,(20)^2}{0.7} = 27.3 \text{ N s/m}.$$

The driver's peak displacement volume (V_D) is computed using the equation from Table 2:

$$V_D = 1.08 \cdot \frac{\sqrt{P_{AR}}}{f_3^2} = \frac{1.08\ \sqrt{(0.7)}}{(20)^2} = 2.26 \times 10^{-3} m^3\ (2260\ cm^3).$$

Likewise x_{max} is calculated:

$$x_{max} = V_D/S_D = 2.26 \times 10^{-3}/0.1134 = 2.0 \times 10^{-2} \text{ m (20 mm)}.$$

In the same manner the parameters for the other system types including the Thiele/Small parameters may be computed from the information contained in Tables 1 - 3. The computed information is shown in tabular format in Fig. 2. Fig. 5 shows plots of the small-signal frequency response, maximum continuous acoustic output, and maximum continuous electrical input power of the three analyzed system types.

Comments

Holding cutoff frequency and efficiency constant in this example means that the higher efficiency constant (k_η) of the vented-box systems shows up in Fig. 2 as a decrease of box volume (V_B). The box volumes range from 0.75m^3 (26.4 ft^3) for the closed-box system (C2ND) to 0.15m^3 (5.3 ft^3) for the vented-box system (V6TH), a 5 to 1 range.

Analysis of the data in Fig. 2 for moving mass (M_{MS}) and electromagnetic damping (R_{ME}) clearly shows the tendency of the vented-box drivers to be heavier and more costly than the closed-box driver if cutoff frequency, efficiency, and driver size are specified. The increased cost for the vented-box drivers is due to the smaller enclosure size (V_B) that these driver's must operate in. If a smaller 300 mm (12 in) drivers could have been used (with increased excursion capability) for the vented sixth-order system (V6TH), its cost would have been comparable to the closed-box driver.

Examination of the graph for maximum input power Fig. 5c reveals the common problem of all vented-box systems of low power handling capacity below box resonance (f_B). The closed-box system can handle up to 66 watts at very low frequencies while the vented-box systems taper off to roughly 5 watts in the same range. These input powers are based on not exceeding the linear excursion limits of the drivers. Most drivers however can normally exceed their x_{max} ratings by some two to three times before being damaged which means that their very-low frequency maximum input power ratings before damage (safe operating range) are significantly higher than that shown. Appropriate high-pass filtering should be used with all vented-box systems if appreciable below band energy in the program material is expected.

The vented-box sixth-order system (V6TH) has a good combination of low-frequency performance, small box size, and high-pass filtering which is inherent in its design. If moderately reduced maximum output in the lowest octave of operation (Fig. 5b) is allowable (the maximum system output curve should match the spectral content of the program material), the sixth-order vented system would be a good choice.

Example 2. Specify f_3, V_B, P_{AR}, and x_{max}.

A "bookshelf" loudspeaker system is to be designed with strong response to 30 Hz and capable of 0.2 acoustic watt midband (107 dB SPL reverberent field, R = 200). Diaphragm excursion is to be limited to a moderate value to limit doppler/FM distortion. Power amplifier and driver size are not important.

The following specifications must be met:

f_3	30 Hz
Response:	Near maximally flat acceptable
V_B	0.057 m^3 (2 ft^3)
P_{AR}	0.2 W
x_{max}	6.25 mm (0.25 in)

The system and driver design data which result from these specifications are shown in Fig. 3. Fig. 6 shows the plots of response, maximum output, and maximum input power.

Comments

Specifying cutoff frequency and box volume leaves system efficiency free to float depending on the value of k_η . This is reflected in Fig. 3 in the efficiency (η_o) row as a range of values of 0.18% for the closed-box system (C2ND) up to 0.9% for the vented system (V6TH). Fig. 6a clearly shows the efficiency differences as a function of frequency.

In this example, a specific maximum acoustic output power P_{AR} of 0.2 watt was specified (see Fig. 6b). This means that the different efficiencies of the three types of systems require different input powers to attain the same output. This is indicated in Fig. 6c where the closed-box system (C2ND) requires 112 watts to generate the same acoustic output that the sixth-order vented-box system (V6TH) can do on 22.4 watts.

A further specification of this example is a specific value of maximum excursion to limit distortion. This allows the cone size to vary to maintain the required value of V_D which depends on the power rating constant (k_P), which is higher for the vented systems. In this example, the specifications were met by a 380 mm (15 in) driver for the closed-box (C2ND), a 250 mm (10 in) for the vented-box fourth-order (V4TH), and a 200 mm (8 in) for the sixth-order vented-box system (V6TH).

Examination of the M_{MS} and R_{ME} rows in Fig. 3 indicate an extremely large range of values with the closed-box (C2ND) values on the very-high side and the vented-box sixth-order (V6TH) system values on the very-low side (42:1 ratio of M_{MS}!, 19:1 ratio of R_{ME}!). The closed-box (C2ND) R_{ME} value of 134 N s/m is larger than the largest magnet assembly JBL makes (about 120 N·s/m) and is bordering on the high-end of realizability! In contrast, the vented sixth-order (V6TH) system M_{MS} of 12.9 g and R_{ME} of 7 N·s/m borders on the low end of realizability!

Marketing Considerations

The biggest problem with the sixth-order vented system (V6TH) is the rather anemic looking 200 mm (8 in) driver in the 2 ft^3 box (the marketing department won't like it!). But now lets look at that 380 mm (15 in) driver in the closed-box system (Wow!, we can really sell that one!). Well, maybe we can put a 300 mm (12 in) or 380 mm (15 in) passive radiator with that 200 mm (8 in) driver in the vented sixth-order system and come up with a sellable package (this system is quite close to the Electro-Voice Inc. system described by Newman in /6/).

Example 3. Specify η_o, V_B, $P_{E(max)}$, and x_{max}

A "sub-mini" compact loudspeaker system is to be designed containing a built-in low-power 20 watt amplifier. The system must be able to generate continuous levels of 104 dB (about 0.1 acoustic watt) in the reverberent field of a typical living room (85 m^3 and R = 200 sabins). Cone excursion is to be limited to 5 mm peak to peak and driver size is not important. Low-end cutoff frequency is for the most part unimportant but must be lower than 100 Hz. These requirements lead to specifications of:

f_3	$<$ 100 Hz
Response:	Near maximally flat acceptable
V_B	0.0142 m^3 (0.5 ft^3)
η_0	0.5% (= 0.1/20 x 100%)
$P_{E(max)}$, P_{ER}	20 W
P_{AR}	0.1 W

The system and driver design data resulting from these specifications are shown in Fig. 4. Plots of frequency response, maximum output, and maximum input are shown in Fig. 7.

Comments

Specifying efficiency and box volume in this example allows f_3 to float so that the efficiency equation eq. (5) is satisfied. Cutoff frequencies range from 67 Hz for the closed-box system down to 39 Hz for sixth-order vented system (Fig. 7a). Eventhough the vented systems have lower response than the closed-box system their moving mass and magnet requirements are less (Fig. 4). Due to the specified x_{max} value the closed-box system uses a 250 mm (10 in) driver while the vented-systems require a smaller 200 mm (8 in) driver.

The lower box resonance frequencies of the vented systems push their displacement limit frequencies down also. Fig. 7c indicates that the systems can handle full rated thermal input power of 20 watts down to: 67 Hz, 48 Hz, and 34 Hz for the C2ND, V4TH, and V6TH systems respectively. The vented sixth-order system can handle full rated power a full octave below the closed box system.

9. CONCLUSION

Equations have been presented that allow computation of the direct-radiator loudspeaker driver's electromechanical parameters directly from system specifications. These equations speed up the design process of drivers for closed-box and vented-box loudspeaker systems and give the designer new insight into system and driver relationships and trade offs. The derived relationships indicate that the vented-box system is extremely economical when compared to the closed-box system when the systems are specified for a particular low-frequency cutoff, enclosure volume, maximum acoustic output, and driver excursion.

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TABLE 1.

SELECTED SYSTEM PARAMETERS

System Parameter	System Type: Closed-Box C2ND	Vented-Box V4TH	Vented-Box V6TH
k_3 ($=f_S/f_3$)	0.433	1.0	1.0
α ($=V_{AS}/V_B$)	5	1.06	2.12
Q_T	0.308	0.40	0.30
Q_{TC}	0.754	----	----
H ($=f_B/f_S$)	0	1.0	1.0
k_η	1.17×10^{-4}	2.34×10^{-4}	5.85×10^{-4}
k_P	0.85	6.9	16
Response Type:	C2	B4	C6

Note. 1. All units SI

2. Efficiency constant k_η computed for η_o in %.

3. The vented-box 6th-order (V6th) system type requires the use of an external 2nd-order high-pass filter/equalizer with a Q of 2 (+ 6 dB lift) at a corner frequency equal to box resonance f_B ($= f_S = f_3$ for this alignment) [4 - 6 , 8].

TABLE 2.

THIELE/SMALL PARAMETER DRIVER DESIGN DATA

Small Signal

$$f_S = \begin{bmatrix} 0.433 & C2ND \\ 1 & V4TH \\ 1 & V6TH \end{bmatrix} \bullet f_3$$

$$V_{AS} = \begin{bmatrix} 5 & C2ND \\ 1.06 & V4TH \\ 2.12 & V6TH \end{bmatrix} \bullet V_B$$

$$Q_{TS} = \begin{bmatrix} 0.308 & C2ND \\ 0.40 & V4TH \\ 0.30 & V6TH \end{bmatrix}$$

$$Q_{MS} = \begin{bmatrix} 3.85 & C2ND \\ 5.0 & V4TH \\ 2.175 & V6TH \end{bmatrix}$$

$$Q_{ES} = \begin{bmatrix} 0.334 & C2ND \\ 0.435 & V4TH \\ 0.348 & V6TH \end{bmatrix}$$

$$\eta_o\ (\%) \begin{bmatrix} 1.17\times10^{-4} & C2ND \\ 2.34\times10^{-4} & V4TH \\ 5.85\times10^{-4} & V6TH \end{bmatrix} \bullet V_B\, f_3^{\,3}$$

Large Signal

$$V_D = \begin{bmatrix} 1.08 & C2ND \\ 0.38 & V4TH \\ 0.25 & V6TH \end{bmatrix} \bullet \frac{\sqrt{P_{AR}}}{f_3^{\,2}}$$

Notes: 1. All units are SI, η_o (efficiency) in %.

2. Brackets enclose constant multipliers which depend on system type where: C2ND, V4TH, and V6TH indicate types: Closed-box 2nd-order, vented-box 4th-order, and vented-box 6th-order respectively.

TABLE 3

DIRECT SYNTHESIS EQUATIONS FOR SPECIFIED CUTOFF FREQUENCY AND EFFICIENCY

Small - Signal

$$M_{MS} = \begin{bmatrix} 0.45 & C2ND \\ 0.805 & V4TH \\ 1.007 & V6TH \end{bmatrix} \bullet \frac{S_D^2 f_3}{\eta_o}$$

$$C_{MS} = \begin{bmatrix} 0.297 & C2ND \\ 0.031 & V4TH \\ 0.025 & V6TH \end{bmatrix} \bullet \frac{\eta_o}{S_D^2 f_3^3}$$

$$Bl = \begin{bmatrix} 1.93 & C2ND \\ 3.41 & V4TH \\ 4.26 & V6TH \end{bmatrix} \bullet S_D f_3 \sqrt{\frac{R_E}{\eta_o}}$$

$$R_{ME} = \begin{bmatrix} 3.71 & C2ND \\ 11.63 & V4TH \\ 18.17 & V6TH \end{bmatrix} \bullet \frac{S_D^2 f_3^2}{\eta_o}$$

Large - Signal

$$M_{MS} = \begin{bmatrix} 0.536 & C2ND \\ 0.117 & V4TH \\ 0.063 & V6TH \end{bmatrix} \bullet \frac{P_{AR}}{x_{max}^2 f_3^3 \eta_o}$$

$$C_{MS} = \begin{bmatrix} 0.252 & C2ND \\ 0.217 & V4TH \\ 0.403 & V5TH \end{bmatrix} \bullet \frac{x_{max}^2 f_3 \eta_o}{P_{AR}}$$

$$Bl = \begin{bmatrix} 2.09 & C2ND \\ 1.30 & V4TH \\ 1.07 & V6TH \end{bmatrix} \bullet \frac{1}{x_{max} f_3} \sqrt{\frac{P_{AR} R_E}{\eta_o}}$$

$$R_{ME} = \begin{bmatrix} 4.36 & C2ND \\ 1.69^{*} & V4TH \\ 1.14 & V6TH \end{bmatrix} \bullet \frac{P_{AR}}{x_{max}^2 f_3^2 \eta_o}$$

Note: Refer to notes on Table 2 and comments in Sec. 7: Specific observations Sec. A.

TABLE 4

DIRECT SYNTHESIS EQUATIONS FOR SPECIFIED CUTOFF FREQUENCY AND BOX VOLUME

Small - Signal

$$M_{MS} = \begin{bmatrix} 3891 & \text{C2ND} \\ 3442 & \text{V4TH} \\ 1721 & \text{V6TH} \end{bmatrix} \cdot \frac{S_D^{\,2}}{f_3^{\,2} V_B}$$

$$C_{MS} = \begin{bmatrix} 3.47 \times 10^{-5} & \text{C2ND} \\ 0.74 \times 10^{-5} & \text{V4TH} \\ 1.47 \times 10^{-5} & \text{V6TH} \end{bmatrix} \cdot \frac{V_B}{S_D^{\,2}}$$

$$Bl = \begin{bmatrix} 178 & \text{C2ND} \\ 223 & \text{V4TH} \\ 176 & \text{V6TH} \end{bmatrix} \cdot S_D \sqrt{\frac{R_E}{f_3 V_B}}$$

$$R_{ME} = \begin{bmatrix} 3.17 \times 10^{4} & \text{C2ND} \\ 4.97 \times 10^{4} & \text{V4TH} \\ 3.11 \times 10^{4} & \text{V6TH} \end{bmatrix} \cdot \frac{S_D^{\,2}}{f_3 V_B}$$

Large - Signal

$$M_{MS} = \begin{bmatrix} 4578 & \text{C2ND} \\ 499 & \text{V4TH} \\ 108 & \text{V6TH} \end{bmatrix} \cdot \frac{P_{AR}}{x_{max}^{2} f_3^{\,6} V_B}$$

$$C_{MS} = \begin{bmatrix} 2.95 \times 10^{-5} & \text{C2ND} \\ 5.08 \times 10^{-5} & \text{V4TH} \\ 23.55 \times 10^{-5} & \text{V6TH} \end{bmatrix} \cdot \frac{x_{max}^{\,2} f_3^{\,4} V_B}{P_{AR}}$$

$$Bl = \begin{bmatrix} 193.1 & \text{C2ND} \\ 84.9 & \text{V4TH} \\ 44.1 & \text{V6TH} \end{bmatrix} \cdot \frac{1}{x_{max}} \sqrt{\frac{P_{AR} R_E}{f_3^{\,5} V_B}}$$

$$R_{ME} = \begin{bmatrix} 37.29 \times 10^{3} & \text{C2ND} \\ 7.20 \times 10^{3} & \text{V4TH} \\ 1.94 \times 10^{3} & \text{V6TH} \end{bmatrix} \cdot \frac{P_{AR}}{x_{max}^{\,2} f_3^{\,5} V_B}$$

Note: Refer to notes on Table 2 and comments in Sec. 7: Specific observations Sec. B.

TABLE 5

DIRECT SYNTHESIS EQUATIONS FOR SPECIFIED EFFICIENCY AND BOX VOLUME

Small - Signal / **Large - Signal**

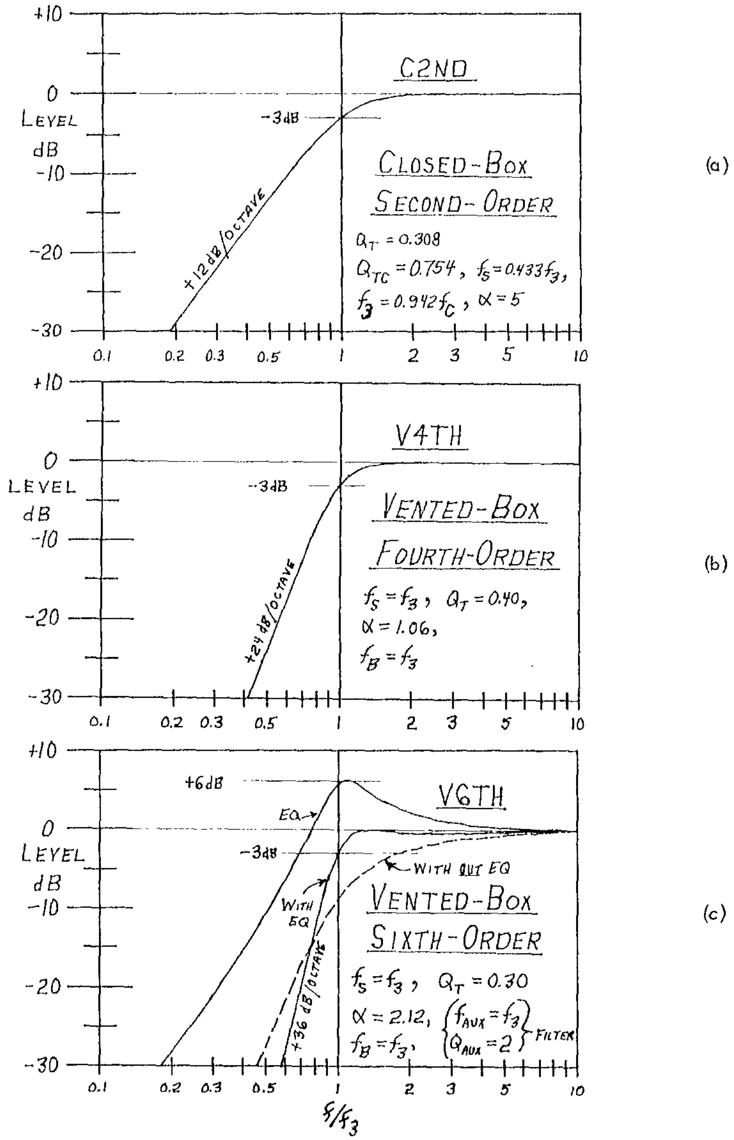
$$M_{MS} = \begin{bmatrix} 9.31 & \text{C2ND} \\ 13.07 & \text{V4TH} \\ 12.04 & \text{V6TH} \end{bmatrix} \cdot \frac{S_D^{\,2}}{\eta_o^{2/3} V_B^{\,1/3}} = \begin{bmatrix} 6.27 \times 10^{-5} & \text{C2ND} \\ 2.73 \times 10^{-5} & \text{V4TH} \\ 3.68 \times 10^{-5} & \text{V6TH} \end{bmatrix} \cdot \frac{V_B P_{AR}}{x_{max}^2 \; \eta_o^2}$$

$$C_{MS} = \begin{bmatrix} 3.47 \times 10^{-5} & \text{C2ND} \\ 0.74 \times 10^{-5} & \text{V4TH} \\ 1.47 \times 10^{-5} & \text{V6TH} \end{bmatrix} \cdot \frac{V_B}{S_D^{\,2}} = \begin{bmatrix} 5.16 & \text{C2ND} \\ 3.52 & \text{V4TH} \\ 4.81 & \text{V6TH} \end{bmatrix} \cdot \frac{x_{max}^2 \; \eta_o^{4/3}}{V_B^{\,1/3} \; P_{AR}}$$

$$Bl = \begin{bmatrix} 39.4 & \text{C2ND} \\ 55.4 & \text{V4TH} \\ 51.0 & \text{V6TH} \end{bmatrix} \cdot S_D \sqrt{\frac{R_E}{\eta_o^{1/3} V_B^{\,2/3}}} = \begin{bmatrix} 0.102 & \text{C2ND} \\ 0.080 & \text{V4TH} \\ 0.089 & \text{V6TH} \end{bmatrix} \cdot \frac{1}{x_{max}} \sqrt{\frac{V_B^{\,2/3} P_{AR} \, R_E}{\eta_o^{5/3}}}$$

$$R_{ME} = \begin{bmatrix} 1550 & \text{C2ND} \\ 3063 & \text{V4TH} \\ 2598 & \text{V6TH} \end{bmatrix} \cdot \frac{S_D^{\,2}}{\eta_o^{1/3} V_B^{\,2/3}} = \begin{bmatrix} 10.44 \times 10^{-3} & \text{C2ND} \\ 6.40 \times 10^{-3} & \text{V4TH} \\ 7.95 \times 10^{-3} & \text{V6TH} \end{bmatrix} \cdot \frac{V_B^{\,2/3} \; P_{AR}}{x_{max}^2 \; \eta_o^{5/3}}$$

Note: Refer to notes on Table 2 and comments in Section 7: Specific Observations Section C.



(a) $Q_T = 0.308$, $Q_{TC} = 0.754$, $f_S = 0.433 f_3$, $f_3 = 0.942 f_C$, $\alpha = 5$

(b) $f_S = f_3$, $Q_T = 0.40$, $\alpha = 1.06$, $f_B = f_3$

(c) $f_S = f_3$, $Q_T = 0.30$, $\alpha = 2.12$, $f_B = f_3$, $\left\{ f_{AUX} = f_3, Q_{AUX} = 2 \right\}$ FILTER

Fig. 1. Small-signal frequency response curves of the three chosen system types whose parameters are shown in Table 1. The curves have been normalized to the system cutoff frequency (f_3) and to their midband efficiency levels. The system types are: a.) closed-box second-order (C2ND), b.) vented-box fourth-order (V4TH), and c.) vented-box sixth-order (V6TH). Refer to Sec. 5 for more information.

EXAMPLE I. PARAMETERS

Specified f_3, η_o, P_{AR}, & S_D

		Parameter	System Type: Closed-Box C2ND	System Type: Vented-Box V4TH	System Type: Vented-Box V6TH
System	Parameters	f_3	←	20 Hz	→
		V_B	0.75 m^3 (26.4 ft^3)	0.37 m^3 (13.2 ft^3)	0.15 m^3 (5.3 ft^3)
		η_o	←	0.7%	→
		f_B	——	20 Hz	20 Hz
		P_{AR}	←	0.7 w	→
Driver Parameters	Thiele/Small	f_s	8.7 Hz	20 Hz	20 Hz
		V_{AS}	3.74 m^3 (132 ft^3)	0.4 m^3 (14.0 ft^3)	0.32 m^3 (11.2 ft^3)
		Q_{TS}	0.31	0.40	0.30
		Q_{MS}	3.9	5.0	2.2
		Q_{ES}	0.33	0.44	0.35
		V_D	2260 cm^3 (138 in^3)	795 cm^3 (48.5 in^3)	523 cm^3 (31.9 in^3)
	Electromechanical	M_{MS}	0.17 kg	0.30 kg	0.37 kg
		C_{MS}	2.0×10^{-3} m/N	2.1×10^{-4} m/N	1.7×10^{-4} m/N
		Bl	13.3 T.m	23.6 T.m	29.4 T.m
		R_E	←	6.5Ω	→
		R_{ME} (∿ cost)	27.3 N·s/m	85.5 N·s/m	133.5 N·s/m
		S_D	←	0.113 m^2 (176 in^2)	→
		Effective Dia.		380 mm (15.0 in)	
		x_{max}	20 mm (0.78 in)	7.0 mm (0.28 in)	4.6 mm (0.18 in)
		P_{ER}, $P_{E(max)}$	←	100 w	→

Fig. 2. System and driver parameters for the three system types of example 1: A sub-woofer system designed to use a 460 mm (18 in) driver with specified cutoff frequency and efficiency. Note that the vented-box drivers have more moving mass and magnet requirements than the closed-box drivers! Note also the wide box volume range for essentially the same performance.

EXAMPLE 2. PARAMETERS

Specified f_3, V_B, P_{AR}, & x_{max}

		Parameter	Closed-Box C2ND	Vented-Box V4TH	Vented-Box V6TH
System	Parameters	f_3	← 30 Hz →	30 Hz	← 30 Hz →
System	Parameters	V_B	← 5.66×10^{-2} m^3 (2 ft^3) →	5.66×10^{-2} m^3 (2 ft^3)	← 5.66×10^{-2} m^3 (2 ft^3) →
System	Parameters	η_o	0.18%	0.36%	0.90%
System	Parameters	f_B	—	30	30
System	Parameters	P_{AR}	← 0.2w →	0.2w	← 0.2w →
Driver Parameters	Thiele/Small	f_S	13.0 Hz	30 Hz	30 Hz
Driver Parameters	Thiele/Small	V_{AS}	0.28 m^3 (10 ft^3)	0.060 m^3 (2.12 ft^3)	0.12 m^3 (4.24 ft^3)
Driver Parameters	Thiele/Small	Q_{TS}	0.31	0.40	0.30
Driver Parameters	Thiele/Small	Q_{MS}	3.9	5.0	2.2
Driver Parameters	Thiele/Small	Q_{ES}	0.33	0.44	0.35
Driver Parameters	Thiele/Small	V_D	539 cm^3 (32.9 in^3)	189 cm^3 (11.5 in^3)	124 cm^3 (7.6 in^3)
Driver Parameters	Electromechanical	M_{MS}	550 g	60 g	12.9 g
Driver Parameters	Electromechanical	C_{MS}	2.7×10^{-4} m/N	4.7×10^{-4} m/N	2.2×10^{-3}m/N
Driver Parameters	Electromechanical	Bl	29.6 T.m	13.0 T.m	6.7 T.m
Driver Parameters	Electromechanical	R_E	← 6.5Ω →	6.5Ω	← 6.5Ω →
Driver Parameters	Electromechanical	R_{ME} (∿ cost)	134.4 N·s/m	26.0 N·s/m	7.0 N·s/m
Driver Parameters	Electromechanical	S_D	8.5×10^{-2} m^2	3.0×10^{-2} m^2	2.0×10^{-2} m^2
Driver Parameters	Electromechanical	Effective Dia.	0.33 m (12.9 in)	0.20 m (7.7 in)	0.16 m (6.2 in)
Driver Parameters	Electromechanical	x_{max}	← 6.35 mm (0.25 in) →	6.35 mm (0.25 in)	← 6.35 mm (0.25 in) →
Driver Parameters	Electromechanical	P_{ER}, $P_{E(max)}$	112 w	56 w	22.4 w

Fig. 3. System and driver parameters for the three system types of example 2: A bookshelf system designed for specific enclosure size, low-end limit, acoustic power output, and diaphragm excursion. Note the very wide differences in driver size, input power, moving mass, and magnet requirements with the vented-box drivers on the low side. If driver size can be allowed to decrease (as in this example) the vented-box drivers can be extremely cost effective.

EXAMPLE 3. PARAMETERS

Specified η_o, V_B, $P_{E(max)}$, and x_{max}

		System Type		
	Parameter	Closed-Box C2ND	Vented-Box V4TH	Vented-Box V6TH
System Parameters	f_3	67.1 Hz	53.2 Hz	39.2 Hz
	V_B	←	1.42×10^{-2} m^3 (0.5 ft^3)	→
	η_o	←	0.5%	→
	f_B	——	53.2 Hz	39.2 Hz
	P_{AR}	←	0.1 w	→
Driver Parameters: Thiele/Small	f_S	29.0 Hz	53.2 Hz	39.2 Hz
	V_{AS}	7.1×10^{-2} m^3 (2.5 ft^3)	1.5×10^{-2} m^3 (0.53 ft^3)	3.0×10^{-2} m^3 (1.06 ft^3)
	Q_{TS}	0.31	0.40	0.30
	Q_{MS}	3.9	5.0	2.2
	Q_{ES}	0.33	0.44	0.35
	V_D	76 cm^3 (4.6 in^3)	43 cm^3 (2.6 in^3)	51 cm^3 (3.1 in^3)
Driver Parameters: Electromechanical	M_{MS}	56.8 g	24.7 g	33.4 g
	C_{MS}	5.3×10^{-4} m/N	3.6×10^{-4} m/N	4.9×10^{-4} m/N
	Bl	14.2	11.1	12.4
	R_E	←	6.5Ω	→
	R_{ME} (∿ cost)	31.0 N·s/m	19.0 N·s/m	23.7 N·s/m
	S_D	3.04×10^{-2} m^2	1.70×10^{-2} m^2	2.06×10^{-2} m^2
	Effective Dia.	0.20 m (7.7 in)	0.15 m (5.8 in)	0.16 m (6.4 in)
	x_{max}	←	2.5 mm (0.1 in)	→
	P_{ER}, $P_{E(max)}$	20 w	20 w	20 w

Fig. 4. System and driver parameters for the three system types of example 3: A sub-mini compact system for use with a low power amplifier targeted for a specific efficiency, box volume, and cone excursion. Note the extension of low-frequency cutoff for the vented-box systems. Even though the vented systems go lower in frequency their drivers still have less mass and magnet requirements (the vented systems use a 200 mm (8 in) driver while the closed-box uses a 250 mm [10 in])

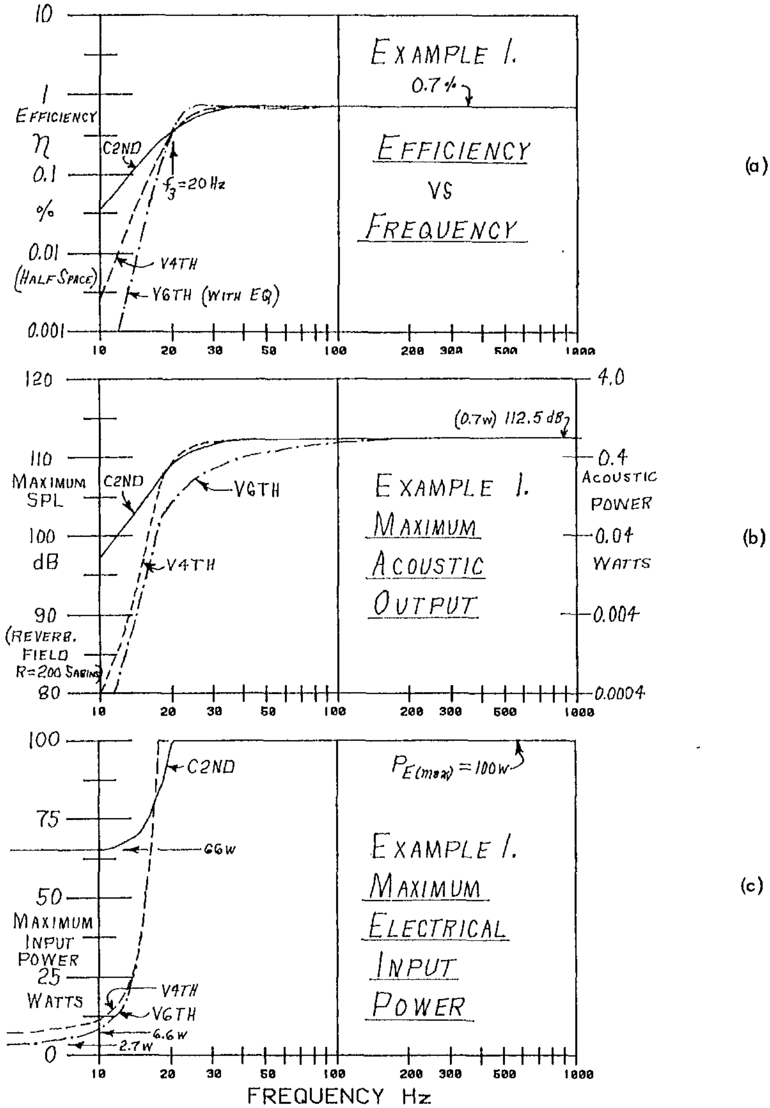


Fig. 5. Display of system response data for example 1: A 20 Hz sub-woofer system using a 460 mm (18 in) driver designed for use with a 100 watt amplifier. Refer to Fig. 2 for system and driver data, and to Sec. 8: example 1 for more details. The curves are: a.) Efficiency vs frequency, b.) maximum continuous acoustic output and SPL (re 20μPa), and c.) maximum electrical input power.

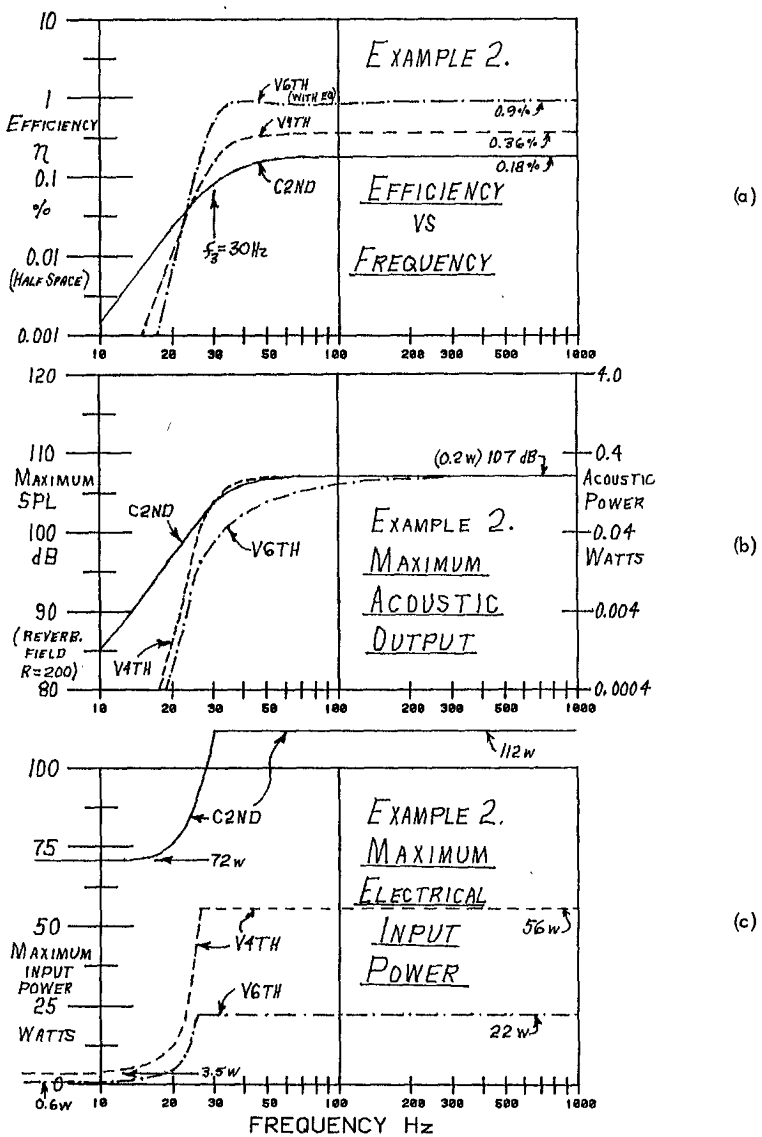
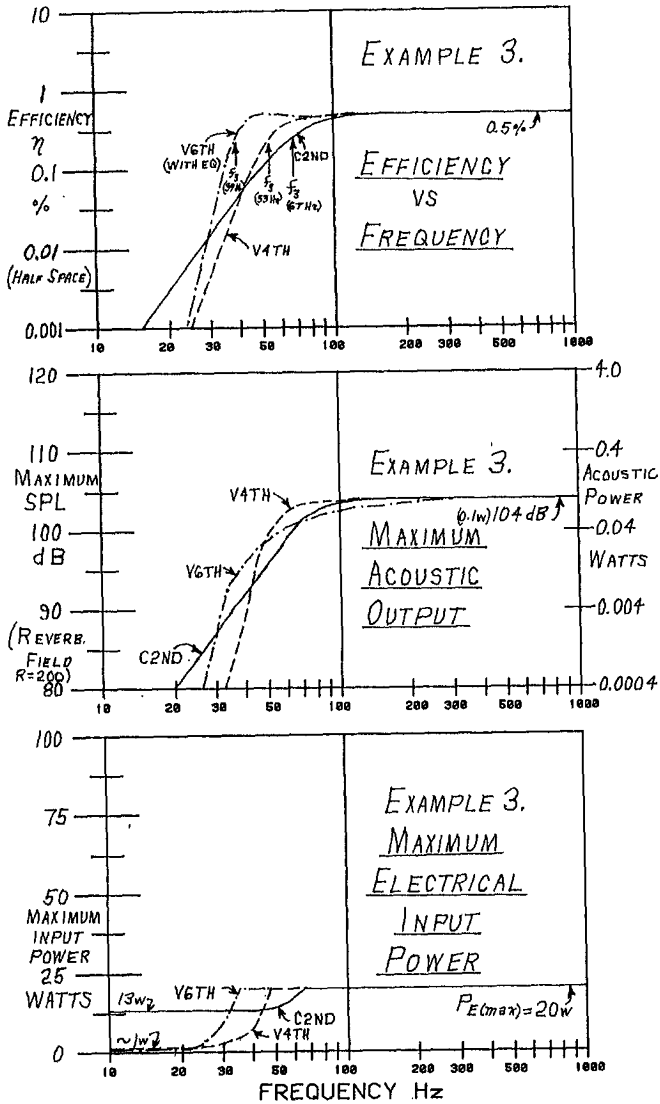


Fig. 6. Plots of system response data for example 2: a 0.057 m^3 (2 ft^3) 30 Hz "bookshelf" system providing 0.2 W of acoustic power. Refer to Fig. 3 for data, and to Sec. 8: example 2 for more information. Plots show: a.) Efficiency vs frequency, b.) maximum output and SPL, and c.) maximum electrical input power.



(a)

(b)

(c)

Fig. 7. Curves of system response data for example 3: a "sub-mini" compact system with specified efficiency (0.5%) and box size 0.0142 m^3 (0.5 ft^3) for use with a 20 watt amplifier. Refer to Fig. 4 for data, and to Sec. 8: example 2 for more details. Graphs indicate a.) Efficiency vs frequency, b.) maximum continuous acoustic output, and c.) maximum electrical input power.