Time-Frequency Display of Electro-Acoustic Data Using Cycle-Octave Wavelet Transforms 4136 (A-4)

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A cycle-octave time-frequency display is created by plotting the magnitude of the wavelet transform, using a Morlet complex Gaussian wavelet, on a log-frequency scale versus time in number of cycles of the wavelet's center frequency. This type of display is quite well suited for plotting the decay response of wide-band systems, such as the impulse response of a loudspeaker, because the time scale is long at low frequencies and short at high frequencies. If the response of typical filters are plotted on this display, the resultant "3-D" responses are independent of the filter's center frequency, i.e. the decay response shape of a particular filter remains the same as it's center frequency is shifted up and down in log frequency.

0. INTRODUCTION

Electro-acoustic data such as the impulse response of loudspeakers and rooms, are often displayed using three-dimensional graphs of amplitude versus time and frequency [1] [2] [3]. These "3-D" displays are more informative than the usual "2-D" displays of magnitude and phase versus frequency, or amplitude versus time such as impulse response or energy time response (ETC). The "3-D" display loosely shows the time response of the system at each frequency or alternately the frequency response of the system at each time. The term loosely is used, because all time-frequency displays suffer from the effects of the uncertainty principle, which states you can't get arbitrarily precise in one domain without simultaneously getting less precise in the other, the product of the two being a constant.

As currently implemented, most "3-D" displays suffer from three disadvantages when applied to wide-bandwidth electro-acoustic data such as the impulse response of loudspeakers. These include: 1. problems of subjective interpretation of the "3-D" display, 2. a frequency resolution or bandwidth not matched to the wide-bandwidth data, and 3. a constant display time scale which is too long at high frequencies to properly display short-lived high-frequency effects and too short at low frequencies to display long-lived low-frequency effects. These disadvantages are described as follows.

0.1. Hard to Interpret Display

One of the major disadvantages of all types of "3-D" displays, is the problem of subjective interpretation of the displayed data. Before a time-frequency display can be created, the resolution in one domain or the other must be chosen (the resolution in the other domain is then set through the uncertainty relationships). This is usually done by selecting such parameters as time window size or frequency bandwidth when the graph is generated. These resolution choices have a major impact on the appearance of the graph, and must be known before the graph can be properly interpreted.

Resolution parameters based on human hearing attributes, such as critical bandwidths, would aid interpretation of "3-D" graphs. One-third- or one-sixth octave bandwidth resolutions would more closely matched human hearing characteristics.

0.2. Mismatched Frequency Resolution

Another disadvantage of "3-D" graphs as usually implemented, is that the frequency resolution is constant and independent of frequency. This automatically makes it unsuitable for wide-bandwidth systems such as loudspeaker responses which may cover a 1000 to 1 range in frequency (20 Hz to 20 kHz), and whose frequency responses are usually plotted on logarithmic frequency scales. Constant frequency resolution works well for data plotted on linear-frequency scales, but not for data plotted on log frequency scales. If the resolution is chosen to match the data at higher frequencies, the low-frequency data will be smeared or smoothed in frequency when plotted on a log frequency scale.

What is needed is a frequency resolution (or bandwidth) that is small at low frequencies and large at high frequencies, i.e., a "constant-Q" resolution that is a constant percentage of the analysis' center frequency, rather than a constant resolution independent of frequency. This would match the analysis bandwidth to the data over the whole frequency band.

0.3. Mismatched Time Scale

The time scale of typical "3-D" displays is almost always fixed at a constant value independent of frequency, and is usually chosen to match mid- and high-frequency effects. For loudspeaker impulse data, time scales of 2 to 10 ms are common. Because loudspeaker low-frequency impulse behavior is always longer than these short time scales, the behavior of the loudspeaker at low frequencies is severely truncated by the display and as a result is not shown.

A time scale is desirable that is short at high frequencies and long at low frequencies, i.e. a time scale inversely proportional to frequency so that the time scale represents a constant number of cycles of the indicated frequency. This is an optimal match to real-world constant-Q physical processes which inherently ring down quicker at high frequencies than low frequencies.

0.4. Wavelets To The Rescue

Wavelet theory provides a rich set of tools for analyzing wideband systems and signals. The wavelet transform inherently provides capability for analyzing data that covers a wide frequency range because the analysis allows signals to be broken down into different frequency ranges, and then studies each range with a resolution matched to its scale. A high-frequency component is studied with a short-time analysis window, while a low-frequency component is studied with a long analysis window.

This paper describes an application of Morlet's cycle-octave transform [4] (called here a cycle-octave wavelet transform) to the time-frequency display of electro-acoustic impulse data. Morlet's cycle-octave wavelet transform uses a complex Gaussian as the mother wavelet. When the magnitude of the transform is plotted on a three-dimensional display of log frequency versus time in number of cycles of the wavelet's center fundamental (as Morlet specifies), the resultant display is very well-matched to wideband systems.

1. OFTEN-USED TIME-FREQUENCY DISPLAYS

Two different types of "3-D" time-frequency displays are commonly used to display the impulse response of loudspeakers and electro-acoustic systems: the windowed or short-time Fourier transform [5], and the cumulative spectral decay plot [1][2] [3].

1.1. Windowed or Short-Time Fourier Transform

The standard Fourier transform,

$$F(\omega) = \frac{1}{\sqrt{2\pi}} \int f(t) e^{-j\omega t} dt, \qquad [1]$$

where

$F(\omega)$ = Fourier transform of $f(t)$, a complex function of frequency
$f(t)$ = signal to be transformed, a function of time
j^2 = -1,

is time blind. It gives the frequency content or spectrum of the signal considering it as a whole, and yields no information about when certain frequencies occur.

Time localization can be achieved by first windowing the signal $f(t)$, so that a well-localized piece of $f(t)$ is transformed into the frequency domain. This is called the windowed or short-time Fourier transform [5]:

$$F(\omega,\tau) = \int f(t) g(t-\tau) e^{-j\omega t} dt, \qquad [2]$$

where

$F(\omega,\tau)$ = short-time Fourier transform of $f(t)$, a complex function of frequency and time
$g(t)$ = window function (a smooth and localized low-pass function),

which is a standard technique for gaining time-frequency localization. The window is shifted down the time record, and successive Fourier transforms are taken at each time point. Note that the window function $g(t)$ has a constant time width and does not depend on frequency, i.e., its width at high frequencies is the same as it is at low frequencies.

When the windowed Fourier transform is applied to wideband data, such as audio or acoustic information that often covers three decades (20 Hz to 20 kHz) or more and is usually plotted on logarithmic frequency scales, it is ill suited because the window is of fixed size in both time and frequency. If the window is chosen to be of proper size so that its frequency bandwidth in the center of the frequency range is correct (say one-third octave), the window will be to narrow at high frequencies and to wide at low frequencies. If plotted on log frequency scales, the data will appear much to smoothed and spread out at low frequencies, while simultaneously having to much detail at high frequencies. As usually plotted with a constant time scale, long low-frequency ring downs will be truncated.

1.2. Cumulative Spectral Decay

The cumulative spectral decay is a special form of the windowed Fourier transform applied specifically to impulse responses, where the window is a step function [2], [3] or a smoothed step function:

$$C(\omega,\tau) = \int h(t) U(t-\tau) e^{-j\omega t} dt \qquad [3]$$

where

$C(\omega,\tau)$	= cumulative spectral decay of $h(t)$, a complex function of frequency and time
$U(t)$	= unit step function
$h(t)$	= impulse response of system.

As τ increases, less and less of the start of the impulse response is converted to the frequency domain. At large delays, only the tail of the impulse response is converted to the frequency domain. Note that when $\tau = 0$, the cumulative spectral decay is equal (within a constant) to the Fourier transform of the impulse response (the steady-state frequency response) $C(\omega,0) = \sqrt{2\pi}\, F(\omega)$. Effectively, the cumulative spectral decay can be interpreted as a windowed Fourier transform of the impulse response, with a time dependent window that is very wide at low delays and shortens as delay increases.

The cumulative spectral decay suffers from some of the same problems as the windowed Fourier transform. Typically, the cumulative spectral decay data is plotted with a logarithmic frequency scale and a constant time scale in the range of 2 to 10 ms. As with the windowed Fourier transform, long low-frequency ring downs will be truncated due to the constant time scale.

2. WAVELET TRANSFORMS

The wavelet transform provides a very-powerful time-frequency expansion that is very well matched to the response of wideband systems. It inherently provides wide-bandwidth narrow time windows at high frequencies, and narrow-bandwidth wide time windows at low frequencies. The transform thus provides optimum time-frequency localization over very wide frequency ranges.

The term "wavelet" was first popularized by Morlet et al. in a series of papers, in the early 80's, which described application to geophysical exploration [4] [6] [7]. Although wavelet theory and research has only been around for little over a decade, the techniques tie together research in many different and diverse disciplines including engineering, physics, and pure and applied mathematics, that goes back over 35 years.

2.1. Continuous Wavelet Transform

The wavelet transform decomposes a time signal into a set of bandpass functions (an oscillatory function with zero average value) called "wavelets," which are time scaled and shifted versions of a "mother wavelet" $\psi(t)$. The continuous wavelet transform is given by [8] [9] (the bar indicates complex conjugation),

$$W(a,\tau) = |a|^{-1/2} \int f(t)\, \overline{\psi\left(\frac{t-\tau}{a}\right)}\, dt \qquad [4]$$

where

$W(a,\tau)$	= wavelet transform of $f(t)$, either real or complex
$\psi(t)$	= mother wavelet, either real or complex
a	= scale value or dilation parameter (effectively sets frequency of wavelet), $a > 1$ corresponds to scaling up (widening) the mother wavelet, and $a < 1$ corresponds to scaling down (narrowing) the wavelet
τ	= time shift or delay.

It is assumed that $\psi(t)$ satisfies the admissibility condition

$$\int \psi(t)dt = 0, \tag{5}$$

i.e., the mother wavelet must have an average value of zero or equivalently a DC value of zero.

As Daubechies observes (with appropriate changes in variables to agree with those in this paper) [9, p. 3];

"The difference between the wavelet and windowed Fourier transforms lies in the shapes of the analyzing functions $g(\omega,\tau) = g(t-\tau)e^{-j\omega t}$ and $\psi(a,\tau) = \psi\left(\frac{t-\tau}{a}\right)$. The functions $g(\omega,\tau)$ all consist of the same envelope function g, translated to the proper time location, and "filled in" with higher frequency oscillations. All the $g(\omega,\tau)$, regardless of ω, have the same width. In contrast, the $\psi(a,\tau)$ have time-widths adapted to their frequency: high frequency $\psi(a,\tau)$ are very narrow, while low frequency $\psi(a,\tau)$ are much broader. As a result, the wavelet transform is better able than the windowed Fourier transform to "zoom in" on very short-lived high-frequency phenomena, such as transients in signals."

2.2. Equivalence of Wavelet Transform and Tone Burst Response

As an aid to understanding the wavelet transform as a tool for analyzing systems, this section shows the equivalence between the wavelet transform of a systems impulse response and the tone burst response of a system, where a time-reversed version of the wavelet is used as the tone-burst excitation signal.

The convolution of two functions $f_1(t)$ and $f_2(t)$ is given by [10, p. 80],

$$f(t) = f_1(t) * f_2(t) = \int_{-\infty}^{\infty} f_1(\tau) f_2(t-\tau)d\tau. \tag{6}$$

Convolution allows the output $f_{out}(t)$ of a time-invariant linear system to be calculated given the impulse response of the system $h(t)$ and the input $f_{in}(t)$: $f_{out}(t) = f_{in}(t) * h(t)$.

Assuming a real mother wavelet and substituting $t = T$ and $\tau = t'$, the wavelet transform Eq. (4) can be written as,

$$\begin{aligned} W(a,t') &= |a|^{-1/2} \int f(T)\, \psi\left(\frac{T-t'}{a}\right) dT \\ &= |a|^{-1/2} \int f(T)\, \psi\left(-\frac{t'-T}{a}\right) dT \\ &= |a|^{-1/2} f(t') * \psi\left(-\frac{t'}{a}\right) \end{aligned} \tag{7}$$

Equation (7) shows that the wavelet transform is equivalent to convolving the input time function with a scaled and *time reversed* version of the mother wavelet. Likewise, the wavelet transform of a system with impulse response $h(t)$ is given by,

$$W(a,t) = |a|^{-1/2} h(t) * \psi\left(-\frac{t'}{a}\right). \tag{8}$$

This shows that the wavelet transform of a system's impulse response is equivalent to a family of tone-burst responses of the system, where the time-reversed wavelet acts as the tone-burst excitation signal!

2.3. Cycle-Octave Transform

Morlet describes a variation of the standard wavelet transform he calls the cycle-octave transform [4] [6]. Here he changes to a octave-based (base 2) logarithmic scale in the dilation parameter by substituting $a = 2^u$ (where u is not necessarily an integer), and changes the time shift variable τ into a "rescaled time shift" $v = \tau / a$, which corresponds to measuring time in terms of "local cycles" (his terms). With these two changes, the wavelet transform of Eq. (4) appears as,

$$W(u,v) = 2^{-u/2} \int f(t) \overline{\psi\left(2^{-u} t - v\right)} dt. \qquad [9]$$

The rescaled time shift parameter v essentially changes the time scale into one that is normalized to the time width of the scaled mother wavelet ψ. Essentially this makes the time scale short at high frequencies and long at low frequencies. Displaying time-frequency data on a sliding time scale of cycles of the fundamental rather than a constant time scale is not original with Morlet, Suzuki et al. earlier suggests its use for displaying "3-D" tone burst responses of loudspeakers [3, 1978]. In 1979 Fryer and Millwood described a measurement system which plotted the data in this way [11].

Equivalently, the modifications effected by these two changes can be accomplished by a simple display post-processing operation using the standard wavelet transform Eq. (4). This can be done by changing from a linear to a log scale for the dilation parameter (equivalently a log frequency scale), and changing the time scale so that it expands and contracts appropriately so that a constant number of cycles of the wavelets center frequency is displayed on the time axis.

Morlet further specifies the use of a Gaussian-windowed complex exponential (sine/cosine) as the mother wavelet,

$$\psi(t) = e^{-bt^2} e^{j2\pi t} = e^{-bt^2} (\cos 2\pi t + j \sin 2\pi t). \qquad [10]$$

The Gaussian wavelet has an optimum time-bandwidth product, and has the same characteristics in the time and frequency domains.

Eq. (10) specifies a one Hz sine/cosine wave windowed by a variable-width Gaussian window (Morlet actually suggested the reverse, a fixed width window with a variable center frequency). The parameter b sets the width of the window and hence the number of cycles in the main part of the wavelet, which directly effects the bandwidth of the wavelet in the frequency domain. Small b yields a wide window with a narrow frequency bandwidth, while large b yields a narrow window with a wide bandwidth. As Morlet points out, this form of Gaussian wavelet does not meet the admissibility constraint of Eq. (5) (zero average value), but effectively does so for commonly used small b. ($b < 1$ approximately)

If the frequency bandwidth of the mother wavelet is specified in fractions of an octave, i.e., $N = 1$ for an octave, $N = 6$ for one-sixth octave, etc., the parameter b is given by (without proof),

$$b = \frac{\pi^2}{4 \ln \sqrt{2}} (\nabla f)^2 = \frac{\pi^2}{4 \ln \sqrt{2}} \left(2^{\frac{1}{2N}} - 2^{-\frac{1}{2N}} \right) \qquad [11]$$

Figure 1 shows the Gaussian mother wavelet of Eq. (10), along with envelope for a one-sixth octave bandwidth ($N = 6$, $b \approx 0.8229$).

3. PRACTICAL IMPLEMENTATION

Rather than directly implementing the wavelet transform equations of Eq. (4) or (9) and numerically calculating the integrals that generate the time-frequency display, an alternate more straightforward method based on FFT (Fast Fourier Transform) techniques is presented in this section. The method starts with a very wide bandwidth log-spaced frequency response data array, and then calculates the individual time response envelopes by manipulating smaller linear-spaced data arrays in the frequency domain and then converting them to the time domain. The "per cycle" time scale normalization is accomplished automatically.

Measurements of audio systems frequently yield sets of frequency response data that cover extremely wide frequency ranges. Loudspeaker measurements often cover a 3.5 decade frequency range extending from 10 Hz to 30 kHz or wider. Storing and manipulating log-spaced data offers a considerable increase in computational efficiency due to the major reduction in the number of sample points required [12]. Often data originates from measurements taken directly in the frequency domain at log-spaced sample points.

If only linear-spaced frequency-domain data is available (as is generated from such measurement systems based on techniques such as impulse FFT, MLS (Maximal Length Sequence, or TDS (Time Delay Spectrometry, a wide band log-spaced data set can be created by combining through interpolation two or more restricted bandwidth linear-spaced responses. For example: a high-resolution 20 Hz to 20 kHz log-spaced spectrum can be created by combining three linear-spaced frequency responses covering individually 0 to 200 Hz, 0 to 2 kHz, and 0 to 20 kHz.

3.1. The Method

The method emphasizes operations in the frequency domain rather than the time domain. The method starts with a wide-bandwidth logarithmically sampled (rather than a linear sampled) magnitude-phase response in the frequency domain. It then calculates the time response of the system at discrete log-spaced frequency points (usually at one-third- or two-third-octave intervals) by multiplying spectrums in the frequency domain and then converting back to the time domain using the FFT. The time envelope (magnitude) is then calculated from the complex time data.

3.2. The Procedure

The following steps outline a practical procedure to create a "3-D" time-frequency display based on the cycle-octave wavelet transform. Implementation details and suggested data structure sizes have been added for clarity.

1. Create a very-wide bandwidth log-sampled frequency response.

Form a very-wide bandwidth log-sampled complex (magnitude-phase) frequency response with a point density (sample points per decade) high enough to properly capture the details of the response. Refer to [12, Eq. (29) or Fig. 13] to determine the proper point density for a specific maximum Q and dynamic range.

A very-wide bandwidth log-spaced working data record must be created covering at least 2 octaves above and below the desired frequency range. Assuming a "3-D" display covering approximately 20 Hz to 20 kHz, this means a working record of at least 5 Hz to 80 kHz. A record covering 5 decades from 1 Hz to 100 kHz is suggested. A sample point density of 1k points per decade is further suggested, which provides for maximum resonant Q's of about 80 for a display dynamic range of 60 dB. The resultant working record is thus 5k points.

2. Calculate spectrum of Gaussian wavelet.

Using a linear-spaced complex data record of 1024 or 2048 points, scaled to -2.56 f_0 to +2.56 f_0 (where f_0 is the center frequency of the scaled wavelet), calculate the spectrum of the Gaussian wavelet (suggest a one-sixth- or one-third-octave bandwidth wavelet).

Force the spectrum to be causal by zeroing out the negative frequency data, and doubling the positive frequency data. The wavelets spectrum is causalized so that when multiplied by the data spectrum and converted back to the time domain, the envelope of the time response can be easily calculated from the complex time data. Causalizing the spectrum and then converting back to the time domain, creates a 90° phase shifted imaginary time response (the sine data), via the Hilbert transform, that can be combined with the real response (the cosine data), to yield the magnitude or envelope time response.

Note that the spectrum of the wavelet need be calculated only once for each "3-D" plot. The spectrum changes that occur from one frequency band to the other are accomplished by just changing the frequency scale of the data.

3. Create linear-sampled frequency response slice.

Starting from the log-sampled complex frequency response of step 1, create (interpolate) a linear-spaced complex data record of size equal to the record of step 2 with the same scaling of $-2.56\,f_0$ to $+2.56\,f_0$. Note that only the positive half of the data record need be filled with data due to subsequent multiplication with the causalized wavelet spectrum.

4. Multiply frequency response slice by spectrum of wavelet.

Create a complex output data record the same size and scaling as the records of steps 2 and 3. Fill the record with the complex product of the of the frequency response slice of step 3 multiplied by the wavelet spectrum of step 2.

5. Convert data to time domain

Using a complex-to-complex inverse FFT, convert the data record of step 4 to the time domain.

6. Calculate envelope of time response

Calculate the envelope (magnitude) of the complex time data calculated in step 5, by computing the square root of the sum of squares of the real and imaginary parts of the time data. Convert data to dB ($20Log$(Magnitude)) if necessary.

7. Repeat steps 3 to 6 at all the desired analysis frequencies

A "3-D" plot covering 20 Hz to 20 kHz with a one-sixth octave bandwidth analysis, stepping every one-sixth octave, would require 61 iterations of steps 3 to 6. All the resultant time envelopes can be displayed side by side, in point by point synchronization, to create the "3-D" time-frequency display. NOTE!! The required "per cycle" time normalization of Sec. 2.3 and Eq. (9) is *automatically accomplished* by the outlined process. The time scale automatically is short at high frequencies and long at low frequencies so as to display a constant number of cycles of the wavelets center frequency. This is due to the changing frequency scale from band to band of the data record in steps 2 and 3.

4. APPLICATION

This section illustrates several examples of the use of the cycle-octave wavelet transform. The transform is first applied to several simulated loudspeaker system responses illustrating systems from a perfect loudspeaker, to a bandpass system containing several high-Q resonances. Secondly, the transform is applied to measurements of an actual loudspeaker. In each situation, the magnitude and phase of the system's frequency response is shown first, and then two forms of the time-frequency display using the cycle-octave wavelet transform: a standard side-view "3-D" elevation view, and a "2-D" contour diagram with contour lines at 10 dB intervals.

4.1. Simulations

Several loudspeaker system responses are simulated in this section: 1. a perfect system with infinite bandwidth and zero phase, 2. a bandpass minimum-phase system, 3. a delayed bandpass system, and 4. a minimum-phase bandpass system containing four identical high-Q resonances equally distributed in log frequency. A typical loudspeaker reference level of 90 dB (one-watt/one-meter) sound pressure level (SPL) is assumed in all the simulations, along with a 20 Hz to 20 kHz log frequency range.

Each time-frequency plot covers an approximate range of 10 to 90 dB SPL, an 80 dB dynamic range. Each time-frequency plot was generated using the procedure of Sec. 3.2 with a one-sixth-octave bandwidth mother wavelet whose time envelope extends down to 90 dB (truncated before and after with a width of 20.87 cycles, see Fig. 1), and the time response was calculated every one-sixth octave (61 bands between 20 Hz to 20 kHz). FFT's were calculated using a 1024 point data record which was interpolated to a 64 point data record to output the time response. A wide-bandwidth log-spaced working data array was used which covered a six-decade range from 0.1 Hz to 100 kHz with 800 points/decade (4800 points).

4.1.1. Perfect System

A system with flat frequency and phase response between 5 Hz and 80 kHz with a sensitivity of 90 dB was simulated. The magnitude and phase responses of this system are shown in Fig. 2. The resultant time-frequency responses are shown in Fig. 3.

The time-frequency response (Fig. 3) just traces out the time envelope shape of the wavelet at each frequency, no additional ring-downs are evident.

4.1.2. Band-pass System

A minimum-phase bandpass system response between 40 Hz and 10 kHz (-3 dB), with sixth-order Butterworth high- and low-pass filters was simulated. The magnitude and phase responses of this system are shown in Fig. 4. The resultant time-frequency responses are shown in Fig. 5.

Note very slight widening of response at the bandpass corner frequencies (40 Hz and 10 kHz) due to the slight additional ringdown of the sixth-order high- and low-pass filters.

4.1.3. Band-pass System With Delay

A non-minimum phase system composed of the minimum-phase bandpass system of Section 4.1.2 in cascade with a pure delay of 1.25 ms. This amount of delay corresponds to 25 cycles at 20 kHz. The magnitude and phase responses of this system are shown in Fig. 6. The resultant time-frequency responses are shown in Fig. 7.

The delay skews the time-frequency plots towards later times at high frequencies. The delay skew is not a straight line due to the logarithmic frequency scale and the per-cycle time display. The low-level energy between 5 and 15 kHz and 35 and 100 cycles is spurious, and is due to numeric errors because of undersampling the rapidly changing phase at high frequencies.

4.1.4. Peaky Band-pass System

A minimum-phase bandpass system of Section 4.1.2 in cascade with four 8-dB Q=10 second-order peak filters at 40, 250, 1600, and 10000 Hz. These peak filters add significant ring-down at their center frequencies. The magnitude and phase responses of this system are shown in Fig. 8. The resultant time-frequency responses are shown in Fig. 9.

Note the close similarity of the ring-down tails in the time-frequency response. This time-frequency responses shows that the cycle-octave wavelet time-frequency display is frequency invariant when constant-Q systems are shifted up and down in frequency.

4.2. Live Loudspeaker Measurements

Axial frequency response measurements (2.83 V rms at one meter on axis) were taken on a two-way floor-standing, bipolar, vented-box speaker system with crossover at 1.5 kHz. The system has an 8-in. cone woofer and a 1-in. dome tweeter on the front and back of the system. The magnitude and phase responses of this speaker system are shown in Fig. 10. The resultant time-frequency responses are shown in Fig. 11.

Note the complex time-frequency response with long ring-down tails that extended out in time. Major delayed responses occur in the vicinity of the crossover (1.5 kHz) and at high frequencies.

5. CONCLUSIONS

The cycle-octave wavelet time-frequency display is quite well matched to wide-band systems. This is because of its constant-percentage octave bandwidth analysis, logarithmic frequency scale, and its "per-cycle" time scale that is short at high frequencies and long at low frequencies. On this type of time-frequency display, the time-frequency response of "constant-Q" type resonators remains the same when shifted up and down in frequency. The display is the logical extension to the time-frequency domain of the standard log-frequency response plot. The equivalence of the wavelet transform and a system's tone burst response to a time-reversed wavelet was shown.

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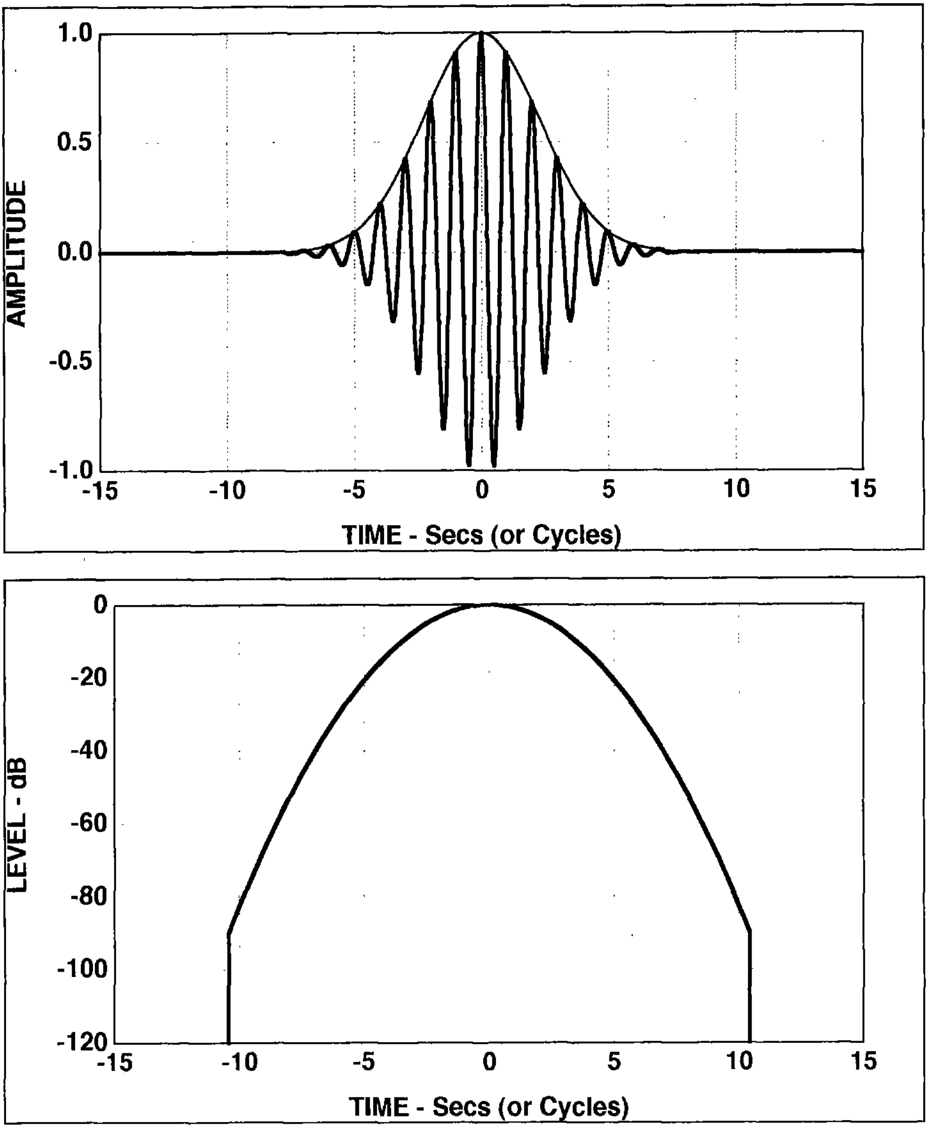

Fig. 1. A complex Gaussian mother wavelet with a one-six octave bandwidth in the frequency domain. The wavelet has been truncated beyond the -90 dB down points of the envelope. Top: *the real part of the wavelet (thick line) along with the wavelets envelope (thin line).* Bottom: the envelope plotted on a log dB scale normalized to the peak of the wavelet.

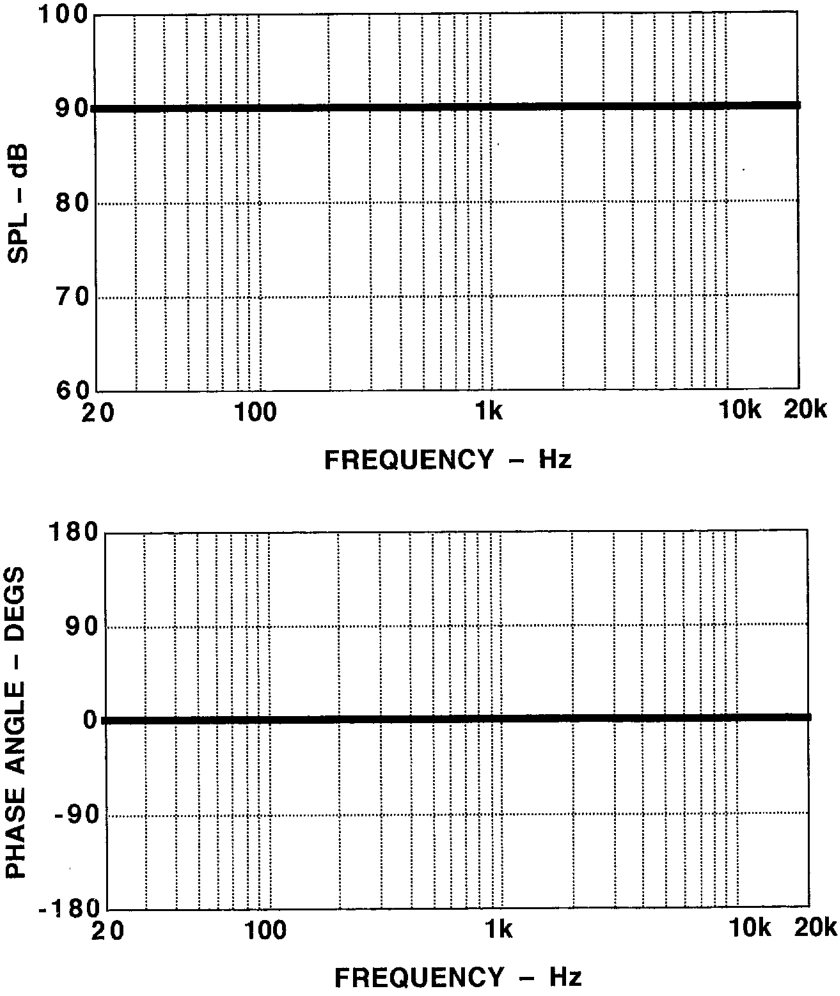

Fig. 2. Responses of a simulated perfect loudspeaker with flat frequency and phase response. Top: magnitude vs frequency response. Bottom: phase vs frequency response.

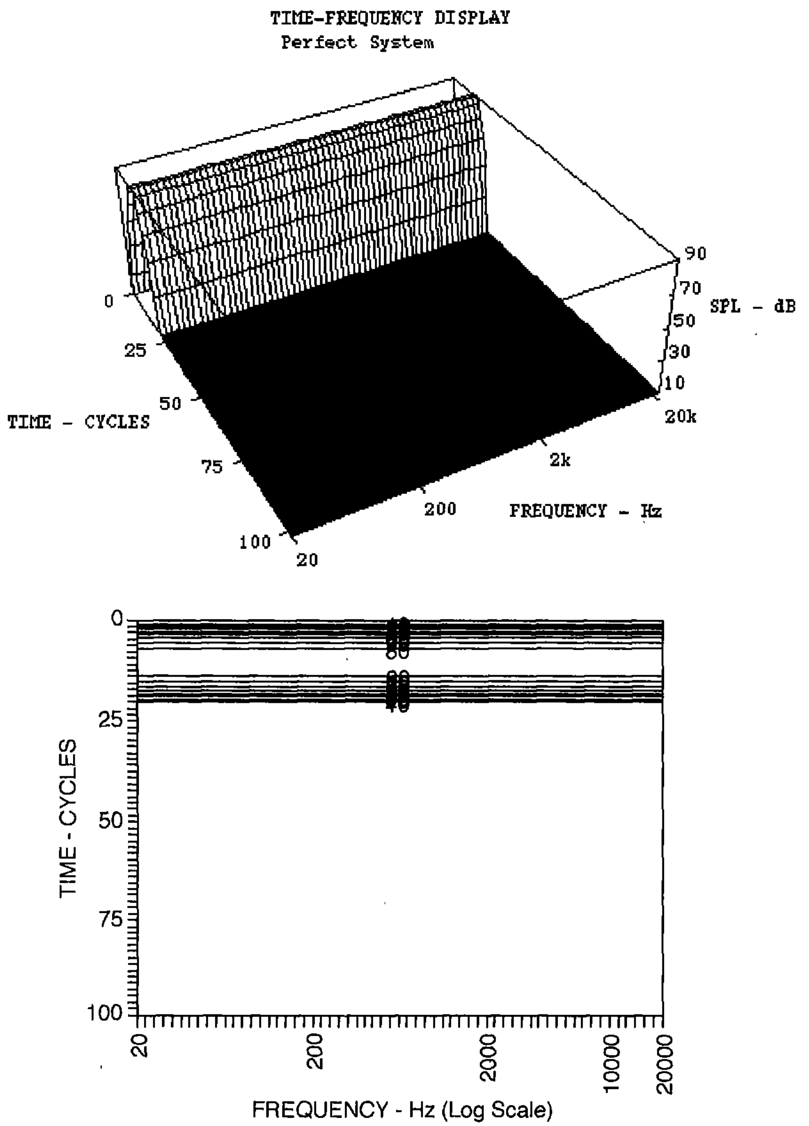

Fig. 3. Time-frequency responses of the perfect loudspeaker of Fig. 2. Top: "3-D" elevation view. Bottom: "2-D" contour diagram.

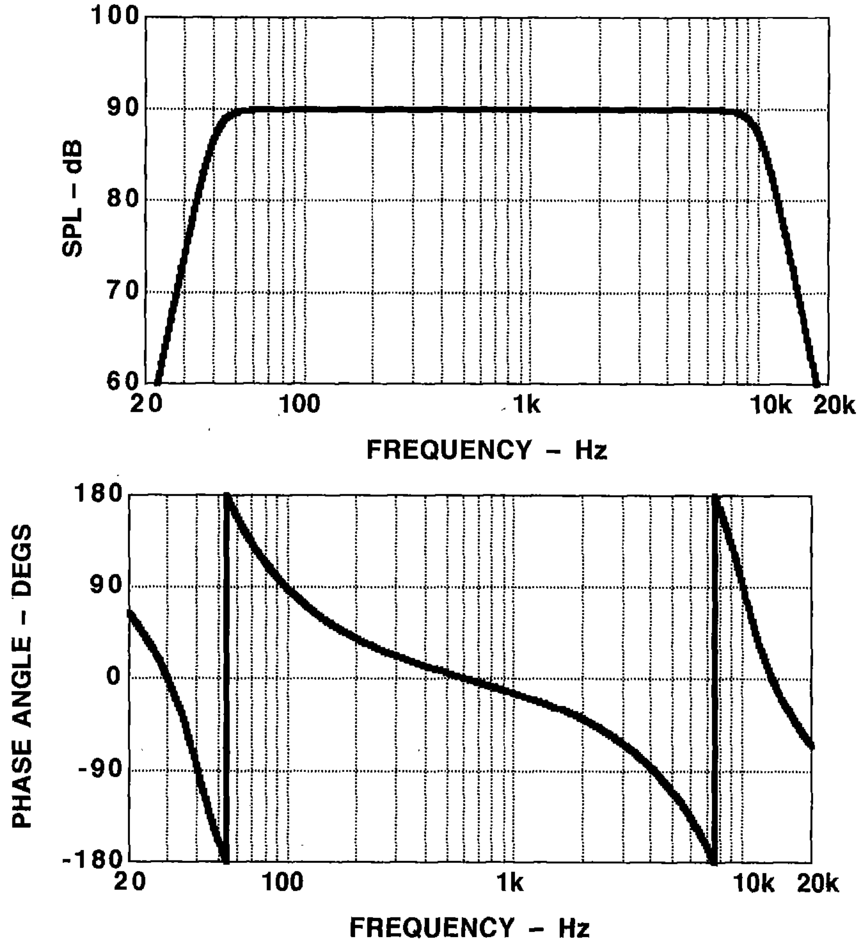

Fig. 4. Responses of a simulated minimum-phase bandpass system. The response has been restricted to 40 Hz and 10 kHz (-3 dB), with sixth-order Butterworth high- and low-pass filters. Top: magnitude vs frequency response. Bottom: phase vs frequency response.

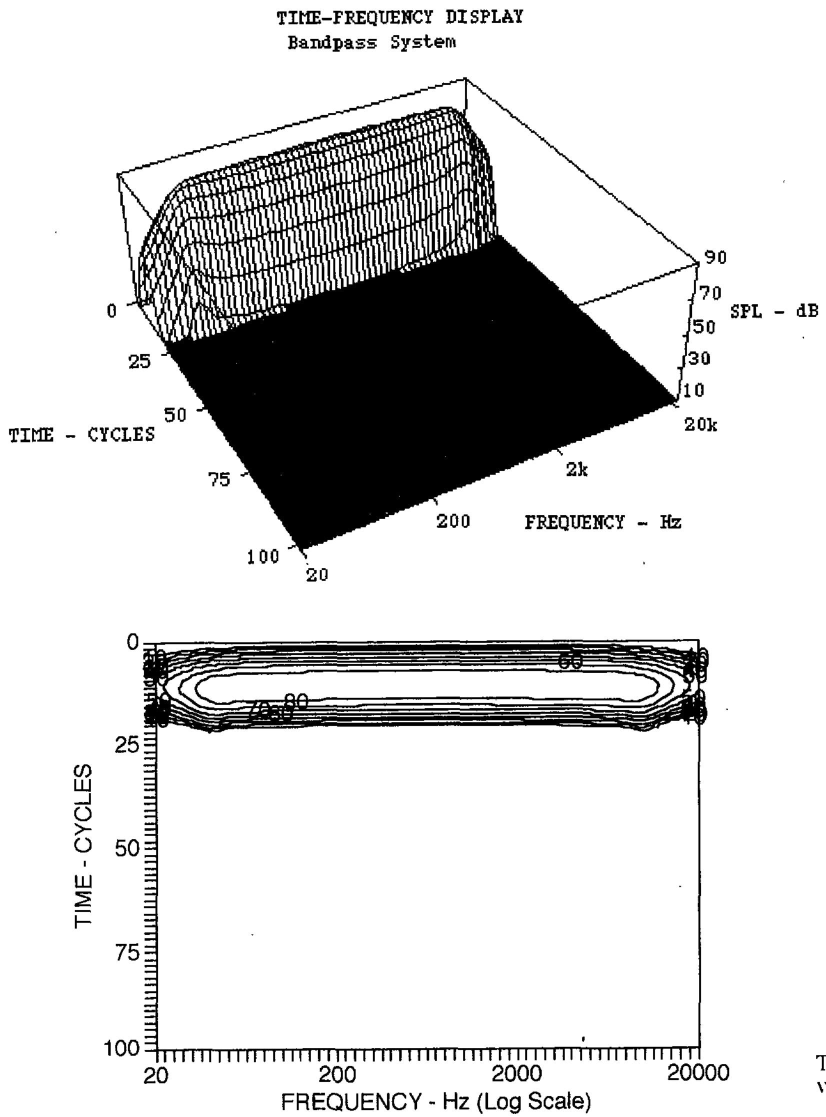

Fig. 5. Time-frequency responses of the bandpass loudspeaker of Fig. 4. Top: "3-D" elevation view. Bottom: "2-D" contour diagram.

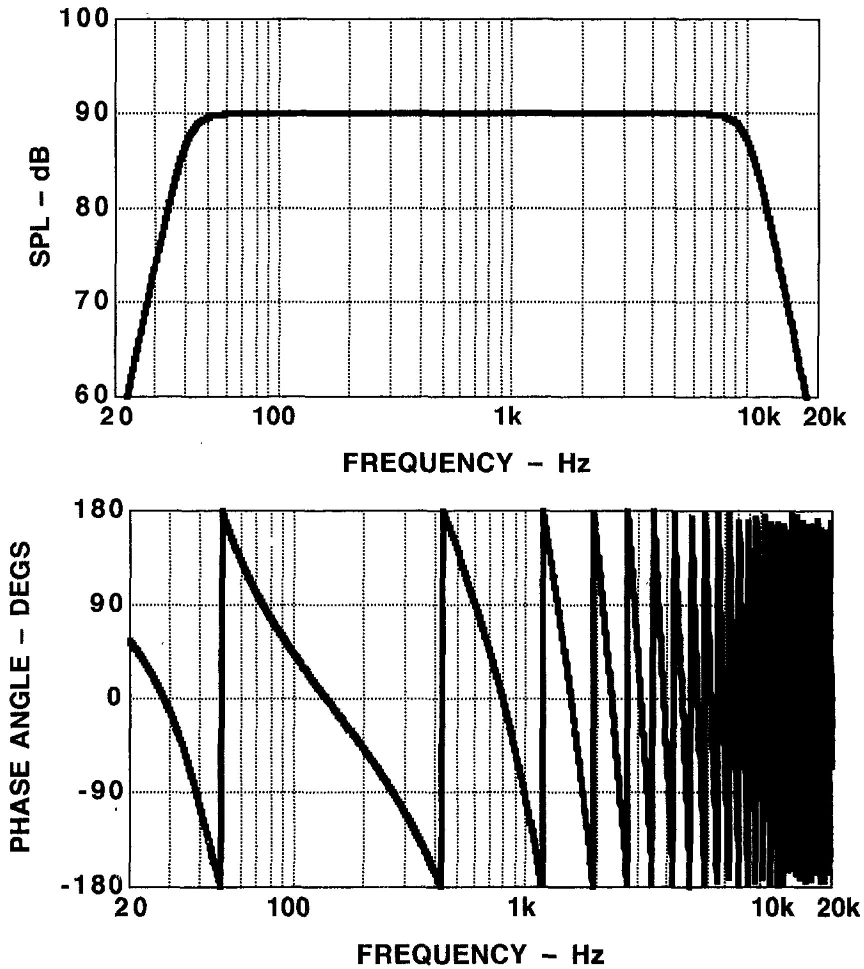

Fig. 6. Responses of a simulated minimum-phase bandpass system in cascade with a pure delay of 1.25 ms. The amount of delay corresponds to 25 cycles at 20 kHz. Top: magnitude vs frequency response. Bottom: phase vs frequency response.

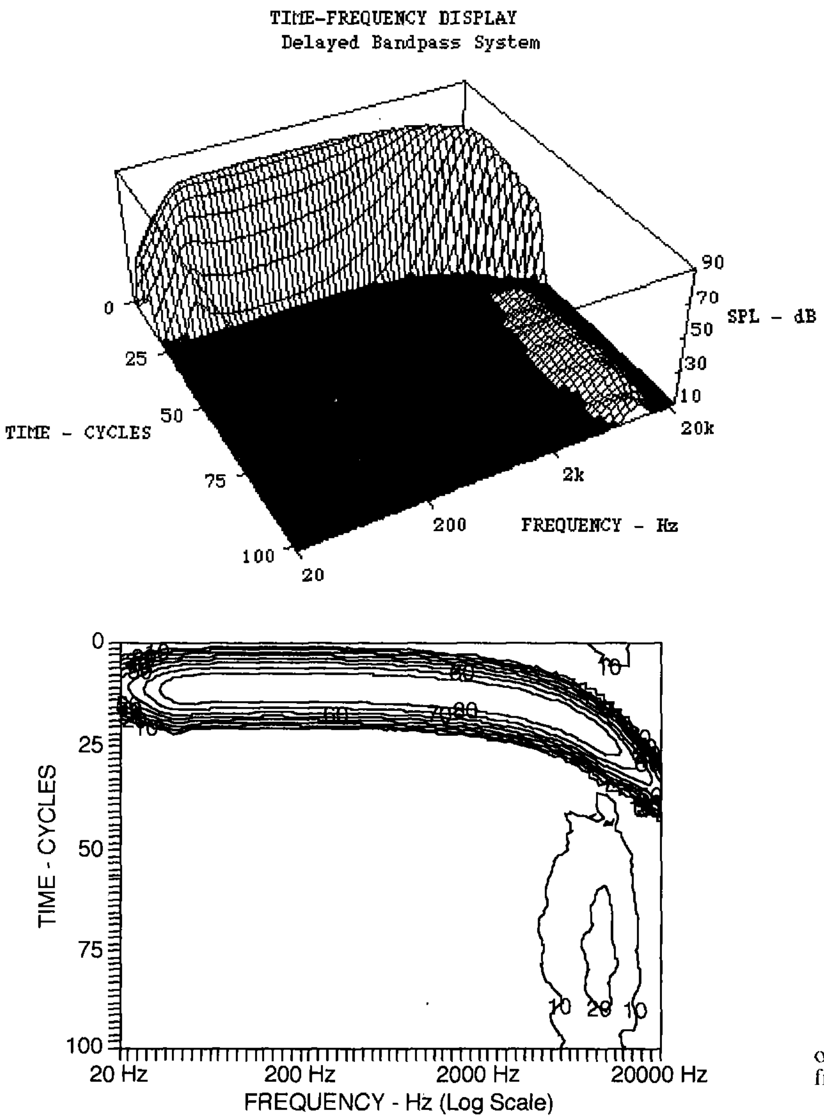

Fig. 7. Time-frequency responses of the delayed bandpass loudspeaker of Fig. 6. Top: "3-D" elevation view. Bottom: "2-D" contour diagram.

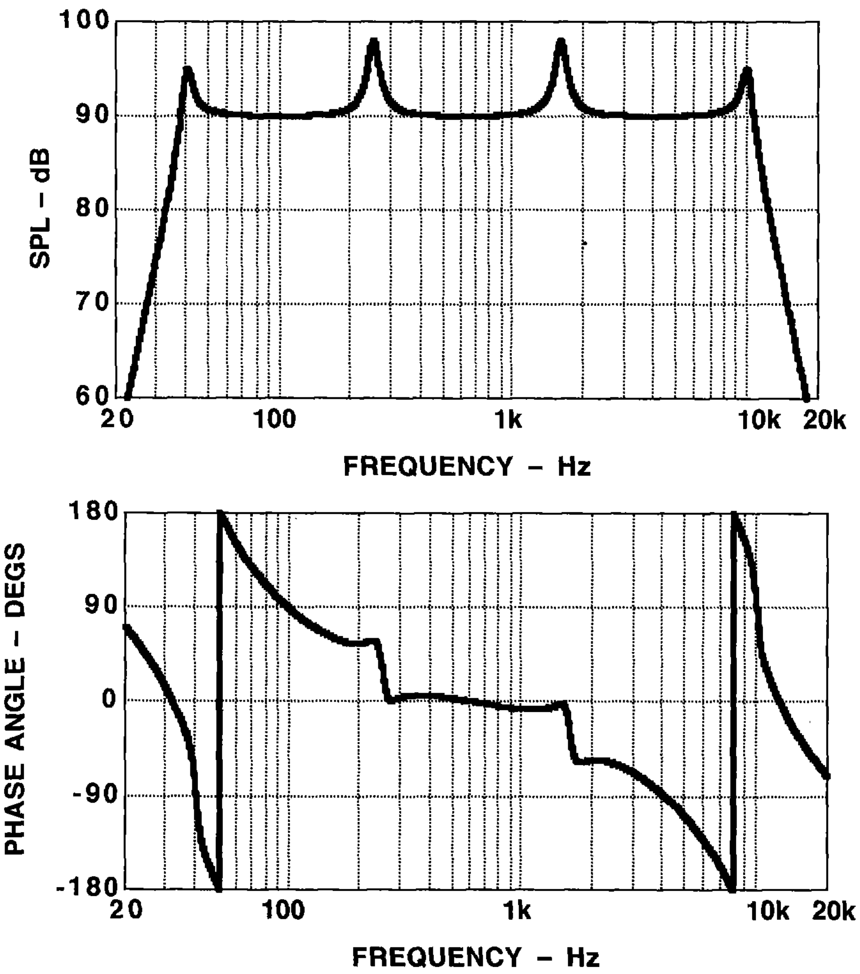

Fig. 8. Responses of a simulated minimum-phase bandpass system in cascade with four 8-dB Q=10 second-order peak filters at 40, 250, 1600, and 10000 Hz. Top: magnitude vs frequency response. Bottom: phase vs frequency response.

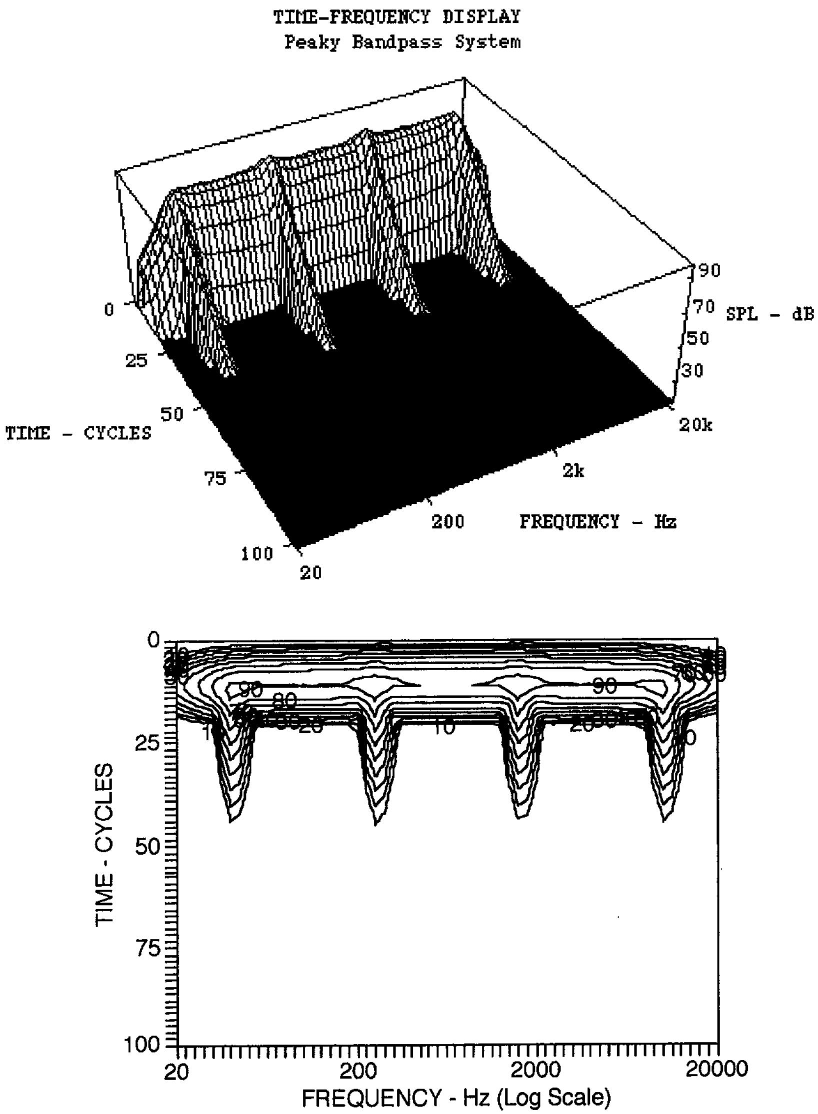

Fig. 9. Time-frequency responses of the "peaky" bandpass loudspeaker of Fig. 8. Top: "3-D" elevation view. Bottom: "2-D" contour diagram. Note the response tails of each resonator. Note also that the time-frequency response of each resonator does not change with shifts in its center frequency.

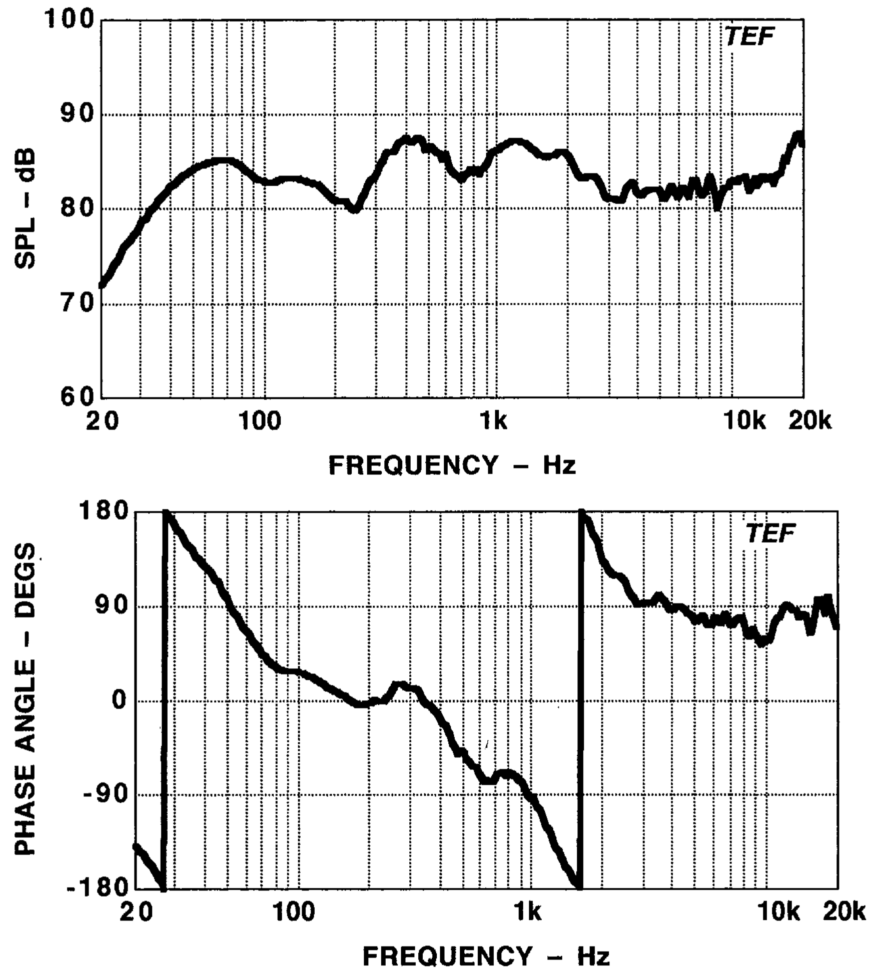

Fig. 10. Measured responses of a two-way speaker system described in Section 4.2. Top: magnitude vs frequency response. Bottom: phase vs frequency response.

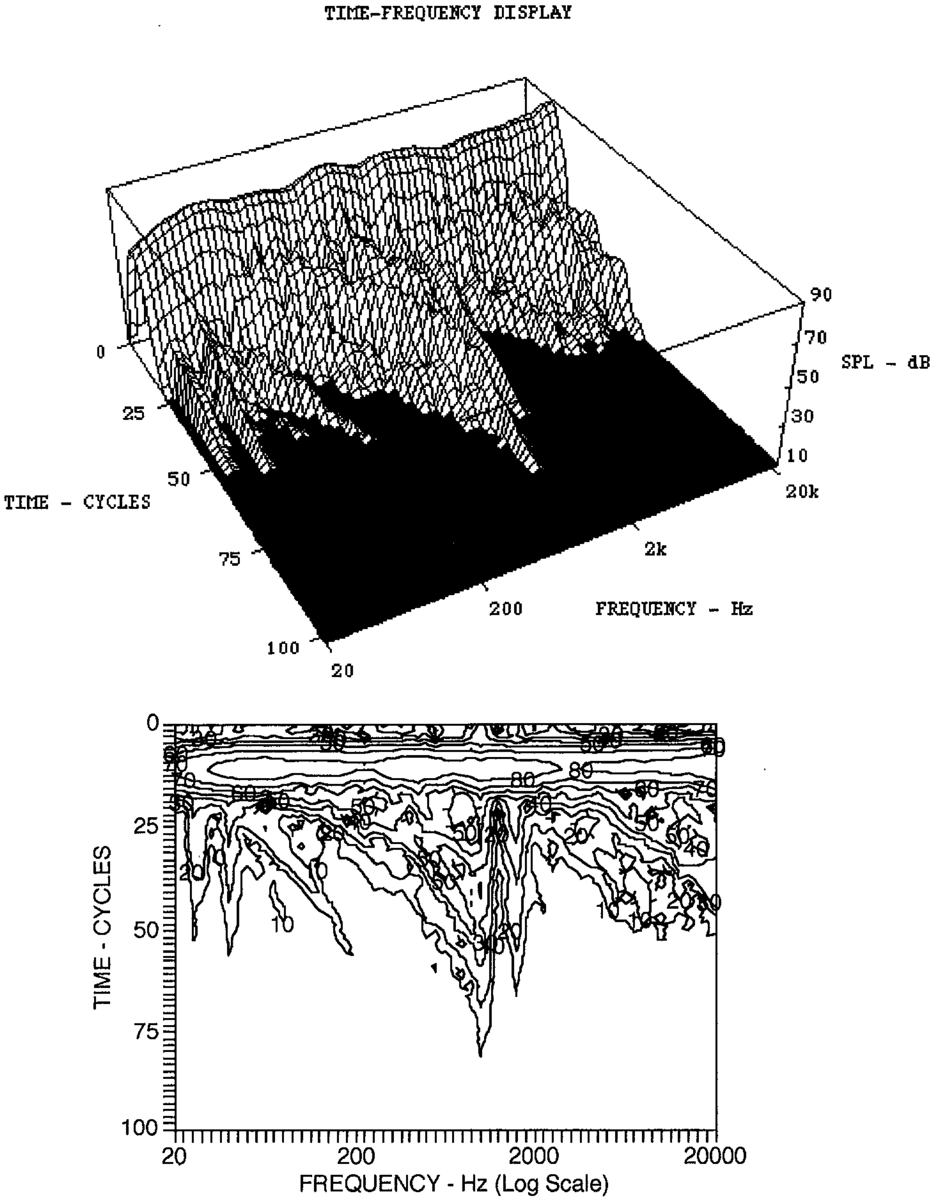

Fig. 11. Time-frequency responses of the loudspeaker of Fig. 10. Top: "3-D" elevation view. Bottom: "2-D" contour diagram. Top: "3-D" elevation view. Bottom: "2-D" contour diagram. Note the rich time-frequency behavior and the extended response at crossover (1.5 kHz) and at high frequencies.